Optimal Bidding Strategies on the Power Market based on the Stochastic Models

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In this paper we concentrate on power producers' strategies on the electricity market with contracts and Asian-type call options for power delivery. We propose stochastic asymmetric supply function equilibrium and Cournot models that are based on the assumption of the stochastic behavior of electricity prices. We demonstrate how in this stochastic case the considered derivatives influence on the power producers' profits. In order to demonstrate theoretical results to the analysis we use the real electricity prices.

Keywords*: *power market, supply function equilibrium, Cournot model, option for power delivery, contract.

I. INTRODUCTION

The transformation of energy market should lead to increasing competitiveness among participants. However specific character of polish energy market (creating regional energy companies) makes that competition among producers is more like oligopoly than perfect competition. Therefore methods of supply function equilibrium or Cournot equilibrium should be used to analyze producers' behavior, [1],[3],[7],[8]. These methods enable better understanding producers' strategies whose play the game on the energy market. The producer is the player and his strategy is maximizing the profit by choosing a bid curve or production level.

In this paper we consider pure and simple producers whose maximize their profits. We assume they can use any forward contracts or Asian-type call options to protect their positions that is represented by the profits. During the recession the question about the profit level was changed into the question about the ruin probability. The probability is based on prices which we treat as random variables with given distribution. As a main result we show the influence of aforesaid derivatives on such probability. In order to demonstrate theoretical formulas to the analysis we use the real electricity prices. We extend the methodology proposed in [1], when the deterministic model was considered as an example of description of power market with derivatives.

II. MODEL SPECIFICATION

We assume there are *m* power producers on the market. The j-th player gives his bid *q_j*, that at time *t* is a function of a price *p_t* and is dependent on two parameters α_j and β_j. The bid curve is a therefore a function given by:

$$q_j : [P_{\min}, P_{\max}] \to [0, U_j],$$

where U_j is a generation capacity for j-th player defined as follows ([1], [3]):

$$q_j(p_t) = \beta_j(p_t - \alpha_j), \qquad j = 1,2,...,m.$$

In the theory, the α_j parameter is interpreted as an intercept of the supply function for j-th player while β_j is a slope of the supply function for player number j.

A complete production cost is a quadratic function given by the following equation:

$$C_j(q_j) = 0.5 d_j q_j^2 + a_j q_j, \qquad j = 1,2,...,m$$

where a_j is an intercept of the marginal cost function for j-th firm and d_j is a slope of the marginal cost function for firm number j.

Moreover the system demand curve is assumed to be:

$$D_t = N(t) - \gamma p_{t,}$$

where *N(t)* is a load-time function, representing the load at time *t* if price were zero, and γ is a slope of the demand to price relationship.

Because for each $j = 1,2,...m$ the function $q_j(p_T)$ is a bid curve for j-th producer at time *T*, then the market cleaning condition is ([2],[7]):

$$\sum_{j=1}^{m} q_j(p_t) = D_t.$$

If we assume the producers can protect profits by forward contracts and Asian-type call options for power delivery during time *[U,T]* (*U*<*T*), then the profit for firm number j at time *T* is given by the following equation,[1], [12]:

$$\pi_{jT} = p_T q_j(p_T) - C_j(q_j(p_T)) + F_{jT}(P_T^F - p_T) + A_{jT}\left(\frac{1}{T-U+1}\sum_{i=U}^{T} p_i - P_T^C\right)^+, \qquad (1)$$

where $(x)^+ = \max(x,0)$, F_{jT} is a forward contract obligation for j-th firm during time period T, P_T^F - forward contract price during time period T, A_{jT} - option obligation for j-th firm for power delivery during time period $[U,T]$ and P_T^C is an option price. In our analysis we assume that the option price at time T is a deterministic value, i.e. it is known at the moment T. The model of profit function given in (1) has the same form as this presented in [1]. The one difference is that the price p_T we treat as a random variable with given probability while in [1] the prices were represented by deterministic values.

Equation (1) we can write as a profit of j-th player in case without the derivatives and the premium from the derivatives, therefore we have:

$$\pi_{jT} = \pi_{jT}^W + F_{jT}(P_T^F - p_T) + A_{jT}\left(\frac{1}{T-U+1}\sum_{i=U}^{T} p_i - P_T^C\right)^+, \quad (2)$$

where $\pi_{jT}^W = p_T q_j(p_T) - C_j(q_j(p_T))$.

III. Stochastic Supply function equilibrium

The first concept of supply function equilibrium (SFE) was used as a way of modeling how competitors (players) could achieve profit-maximizing equilibria in the market place under conditions of uncertain demand [10]. Next, the SFE approach was proposed by Green and Newbery [6] as a model for strategic bidding in competitive spot market for electricity, see also [5]. In our analysis we extend such approach, i.e. the SFE model we treat as a basic model in a power market with contracts and Asian-type options. We also extend the classical methodology of SFE by analyzing stochastic approach of market behavior, that means we treat power prices p_T as random variables with given distribution. The competitors can protect their profit by using derivatives, therefore the ruin probability is smaller than in classical power market.

Using results presented in [1] in the non-stochastic SFE model with contracts and Asian-type options we obtain the following α_j and β_j parameters at equilibrium, i.e. that provide to the maximization of the profit π_{jT} for each $j=1,2,...,m$ (see Lemma 3 in [1]):

$$\beta_j^* = \frac{2}{d_j}$$

$$\alpha_j^* = \begin{cases} a_j + \dfrac{d_j}{2} F_{jT} & if (A) \\ a_j + \dfrac{d_j}{2}\left(F_{jT} - \dfrac{A_{jT}}{T-U+1}\right) & if (B), \end{cases}$$

where the conditions (A) and (B) are as follows:

$$(A): \quad p_T < P_T^C (T-U+1) - \sum_{i=U}^{T-1} p_i$$

$$(B): \quad p_T \ge P_T^C (T-U+1) - \sum_{i=U}^{T-1} p_i. \quad (3)$$

The conditions (A) and (B) follow directly form the definition of profit function of Asian-type options, see [1]. In the stochastic SFE model, similar as in classical approach, we can calculate the profit of j-th player. Because the price p_T is a random variable, therefore the equilibrium profit (i.e. calculated for α_j^*, β_j^* parameters) is also a random variable. The available derivatives only protect the profit for each player therefore in our analysis we discuss the equilibrium profit π_{jT}^{W*}, that is actually the proper income for each competitor. At equilibrium the profit π_{jT}^{W*} has the following form:

$$\pi_{jT}^{W*} = \begin{cases} p_T F_{jT} - C_j(F_{jT}) & if (A) \\ p_T\left(F_{jT} - \dfrac{1}{T-U+1} A_{jT}\right) - C_j\left(F_{jT} - \dfrac{1}{T-U+1} A_{jT}\right) & if (B). \end{cases}$$

A company is in ruin if its total profit in long term goes below zero, because it can not cover its total costs. In short term a company has problems with profitability of production if its total profit plus fixed costs goes below zero. We consider a company activity in long term, which we divided into some one-period games as the introduction to long term analysis. Below we proposed the definition of ruin probability in the stochastic SFE model. In the stochastic SFE model, similar as in the insurance theory, we can calculate the ruin probability at time T. The ruin probability at equilibrium is equal to:

$$\Pr(\pi_{jT}^{W*} < 0) = \Pr(p_T < \min(0.5 d_j F_{jT} + a_j, P_T^C (T-U+1) - \sum_{i=U}^{T-1} p_i))$$
$$+ \Pr(P_T^C (T-U+1) - \sum_{i=U}^{T-1} p_i \le p_T < 0.5 d_j \left(F_{jT} - \frac{1}{T-U+1} A_{jT}\right) + a_j).$$

This ruin probability we can simulate by using the distribution of the price p_T. As an example let us consider the following values of considered parameters:

$$P_T^C = 1, \qquad \sum_{i=U}^{T-1} p_i = 1.$$

Moreover we assume that each players cost is a convex function given by

$$C_j(x) = 0.25x^2 + x, \qquad j = 1,2,...,m.$$

In our simple example we analyze three power producers p_1, p_2, p_3. Because in our discussion we demonstrate how the derivatives, especially the option, influence on the player's profit therefore we assume the contract obligations (F_{jT}) are equal for each competitor and Asian-type option (A_{jT}) obligations for $j=1,2,3$ are as follows:

$$A_{1T} = 30, \qquad A_{2T} = 20, \qquad A_{3T} = 10.$$

On Fig.1 we illustrate the ruin probability in stochastic SFE model for a given parameters. Because many real prices can be described by a log-normal distribution (see Section V) therefore we take the assumption that p_T has a standard log-normal distribution that means $p_T = exp(Y)$, where Y has standard normal distribution. We present the ruin probability as a function of a time period $T\text{-}U\text{+}1$, i.e. time when the power is delivered.

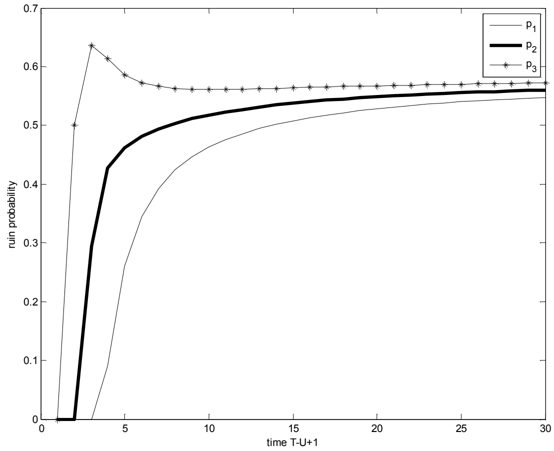

Figure 1. The ruin probability as a function of power delivery time period in stochastic SFE model.

As we expected, for player p_1 we observe the smallest value of ruin probability. This is connected with the largest value of option obligation for this competitor.

In order to demonstrate that the Asian-type option influence on the value of ruin probability on Fig.2 we illustrate this probability in case with contract and Asian-type option in comparison with case without this derivatives. Because for each player p_1, p_2, p_3 we obtain the similar results therefore we only show the comparison for player number 1.

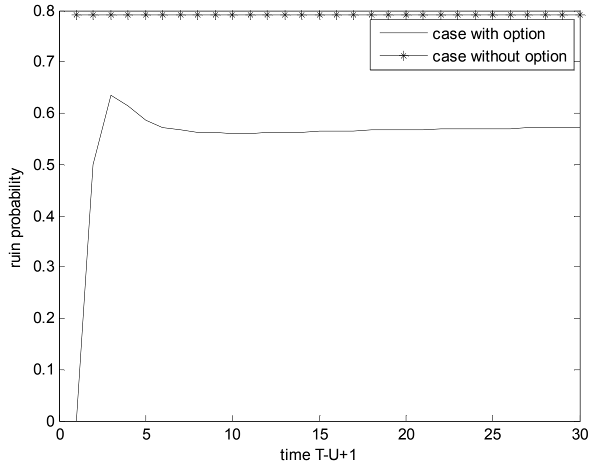

Figure 2. The ruin probability for player p_1 as a function of power delivery time period in stochastic SFE model. Comparison of two cases: with and without option.

In case of stochastic supply function equilibrium for different quantities of Asian-type options we obtained different functions of ruin probability. As we observe, the safest strategy is this of the producer's p_1, i.e. with largest value of Asian-type options. If the producer does not use such derivative he will obtain higher level of ruin probability.

IV. STOCHASTIC COURNOT CMPETITION MODEL

Cournot equilibrium is commonly used as a solution concept in oligopoly models. It is connected with non-cooperative games. Each player makes a decision independently and attempts to maximize his profit by choosing its level of production (quantity). Moreover each competitor does not know production level of other player [11].

Using results obtained in [1], when the non-stochastic Cournot model was analyzed as an example of power market with contracts and Asian-type options, we obtain the following form of profit π_{jT}^{W*} in the stochastic Cournot equilibrium model:

$$\pi_{jT}^{W*} = \begin{cases} \dfrac{\gamma}{2(1+\gamma d_j)^2}\left(p_T - a_j + \dfrac{F_{jT}}{\gamma}\right)\left((p_T - a_j)(2+\gamma d_j) - d_j F_{jT}\right) & if\,(A) \\ \dfrac{\gamma}{2(1+\gamma d_j)^2}\left(p_T - a_j + \dfrac{F_{jT}}{\gamma} - \dfrac{A_{jT}}{\gamma(T-U+1)}\right)\left((p_T - a_j)(2+\gamma d_j) - d_j F_{jT} + \dfrac{d_j A_{jT}}{T-U+1}\right) & if\,(B). \end{cases}$$

Therefore the ruin probability (in sense presented in Section III) at time T in stochastic Cournot equilibrium model is equal to:

$$\Pr(\pi_{jT}^{W*} < 0) = \Pr\left(a_j < p_T < \min\left(\frac{d_j}{2+\gamma d_j} F_{jT} + a_j, P_T^C (T-U+1) - \sum_{i=U}^{T-1} p_i\right)\right)$$
$$+ \Pr\left(\max\left(P_T^C (T-U+1) - \sum_{i=U}^{T-1} p_i, a_j + \frac{A_{jT}}{\gamma(T-U+1)} - \frac{F_{jT}}{\gamma}\right) \le p_T < a_j + \frac{d_j F_{jT}}{(2+\gamma d_j)} - \frac{d_j A_{jT}}{(T-U+1)(2+\gamma d_j)}\right).$$

Similar as in stochastic SFE model we demonstrate the ruin probability in stochastic Cournot equilibrium model. We take the same parameters as in Section III. Moreover, similar as in SFE case, we analyze the ruin probability for three players p_1, p_2, p_3.

On Fig. 3 we compare the ruin probability in stochastic Cournot case for competitors as a function of power delivery time in the period *T-U+1*.

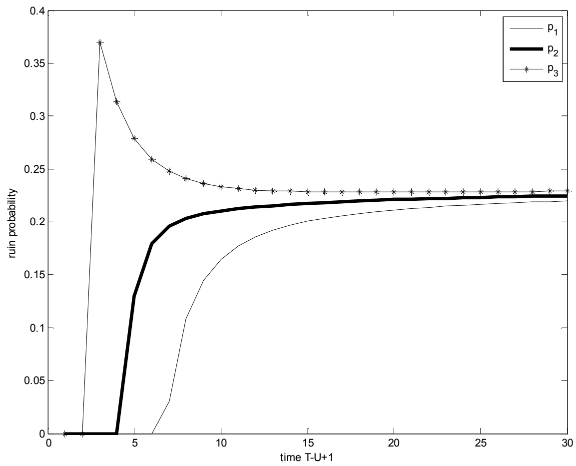

Figure 3. The ruin probability as a function of power delivery time period in stochastic Cournot equilibrium model.

In case of stochastic Cournot equilibrium model for different quantities of Asian-type options we also observed different functions of ruin probability. We observed smaller values of ruin probabilities in case of producer p_1. So producer's p_1 strategy is the safest one.

Similar as in SFE case let us compare two cases, i.e. case with and without derivatives obligation in order to illustrate the option impact on the profit function. Therefore on Fig. 4 we demonstrate such two cases for player p_1.

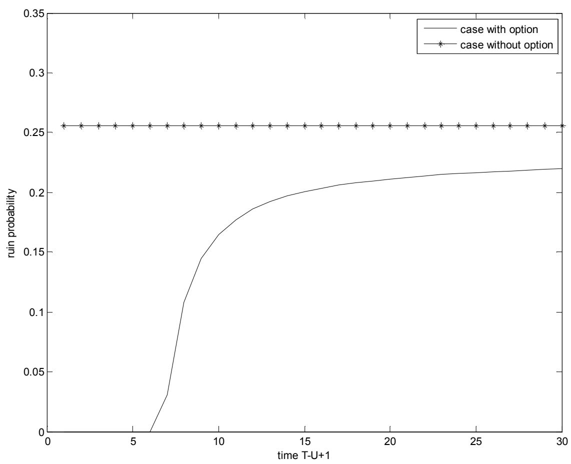

Figure 4. The ruin probability for player p_1 as a function of power delivery time period in stochastic Cournot equilibrium model. Comparison of two cases: with and without option.

Similar as in the previous case we observe the expected behavior, that means in case with Asian-type option we have the smallest value of ruin probability as in case without such derivative.

V. CASE STUDY

In order to demonstrate the theoretical results to the analysis we take the real power price data from Polish Power Exchange. We analyze the averages power prices from day-ahead market (PLN/MWh) from year 2009. Because the data are not stationary therefore to our discussion we take only prices from Thursdays from 4 p.m.

Using the statistical tests based on Kolmogorov-Smirnov statistics we can conclude the data can be described by log-normal distribution. On Fig. 5 we present the kernel estimator of density based on our data in comparison with fitted log-normal density function.

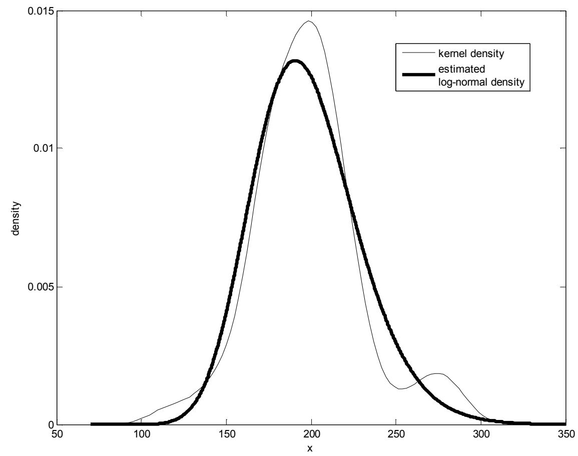

Figure 5. The kernel estimator of density function based on the real power data and the log-normal density with estimated parameters.

On Fig.6 we demonstrate the ruin probability based on the real data in case of stochastic SFE model, whereas on Fig. 7 we show such probability in stochastic Cournot equilibrium model. Similar as in Sections III and IV we analyze the three-players game. Moreover we take the following parameters:

$$P_T^C = 20, \qquad \sum_{i=U}^{T-1} p_i = 20.$$

$$C_j(x) = x^2 + 5x, \qquad j = 1,2,3.$$

$$F_{1T} = 100, \qquad F_{2T} = 100, \qquad F_{3T} = 100.$$

$$A_{1T} = 300, \qquad A_{2T} = 200, \qquad A_{3T} = 100.$$

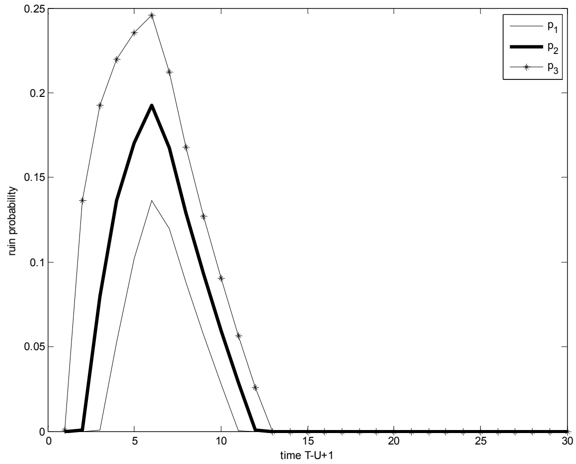

Figure 6. The ruin probability as a function of power delivery time period in stochastic SFE model based on real power data.

Similar as for the theoretical analysis presented in Sections III and IV we observe here the expected behavior. For the player p_1 we have the smallest value of the ruin probability because of the largest value of option obligation.

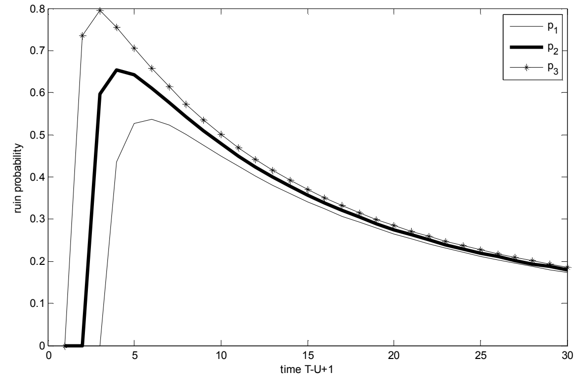

Figure 7. The ruin probability as a function of power delivery time period in stochastic Cournot equilibrium model based on real power data.

We can also compare the ruin probabilities in two considered cases. On Fig. 8, 9 and 10 we demonstrate such comparison for three players p_1, p_2, p_3.

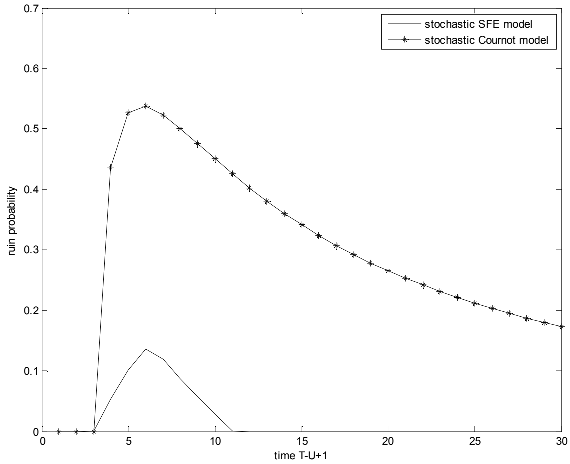

Figure 8. The ruin probability for p_1 in stochastic SFE and Cournot equilibrium model based on real power data.

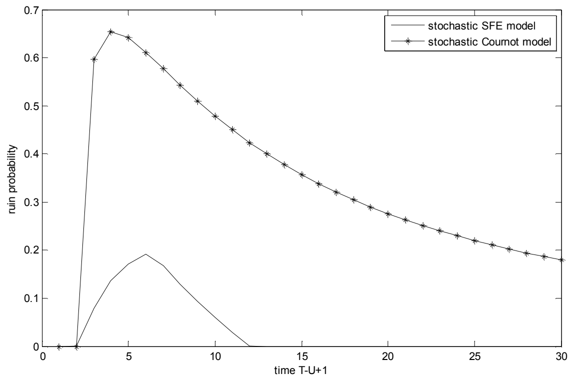

Figure 9. The ruin probability for p_2 in stochastic SFE and Cournot equilibrium model based on real power data.

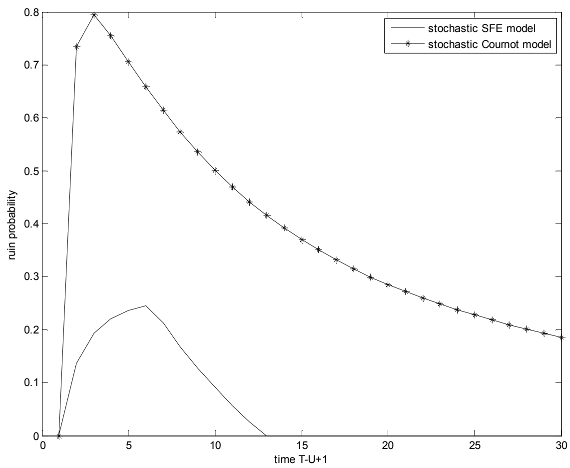

Figure 10. The ruin probability for p_3 in stochastic SFE and Cournot equilibrium model based on real power data.

SUMMARY

In our discussion we treat power producers as players on energy market. Their main aim is to maximize their profits. Under this assumption we showed that the method of stochastic supply function equilibrium as well as the stochastic Cournot equilibrium model can be used to the analysis of power producer's behavior on the energy market with derivatives such as contracts and Asian-type options.

In both stochastic cases for different quantities of Asian-type options we obtained different functions of ruin probability. In simple examples we observed smaller values of ruin probabilities for producer with largest value of Asian-type option obligation (i.e. producer p_1), that suggests the derivative in the considered cases protect the profit.

We presented the connection between value of option obligation and level of ruin probability for both methods: the method of supply function equilibrium and Cournot equilibrium model. We pointed out the Asian-type option has greater influence on level of ruin probability than the contract obligation. We extended the approach proposed in [1], when only deterministic cases of SFE and Cournot models were considered as descriptions of power market with contracts and Asian-type options

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