SETARX Models for Spikes and Antispikes in Electricity Prices

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***Abstract*—This paper discusses two regime-switching TARX models for electricity prices that include spikes, antispikes and microeconomic threshold effects typical of power markets, without using jumps. Preliminary version.**

***Index Terms*—Stochastic processes, time series analysis, power system economics.**

I. Introduction

Each power market has its own idiosyncratic organizational and participants' characteristics, but all power markets show a common set of 'stylized facts' in their electricity price dynamic features, 'facts' very different from those found in the much more studied and much more liquid stock or bond markets. All power markets are built around the necessity of a timely and reliable physical delivery of electricity from producers to users, at socially best prices. Associated to spot or forward physical markets, markets of purely financial derivatives written on the underlying electricity price processes help to manage price or quantity risks. Consequently, good mathematical models for physical electricity price dynamics are necessary for a proper optimization of social welfare in the electricity industry. As it usually happens in research, the first models of electricity prices were built in imitation of stock or bond market models [1]. In continuous time t this means for example the use of geometric brownian motions or continuous AR(1) (Ornstein-Uhlenbeck) stochastic differential equations. Unfortunately, these off-shelf models cannot reproduce power markets stylized facts. Moreover, such models are usually designed and used to describe fundamental markets, in the sense that they are not explicitly driven by exogenous processes other than standard Wiener processes (except maybe when including credit risk). On the contrary, in power markets, in the presence of *tight market conditions* due to *capacity constraints* or *power grid limitations*, exogenous electricity demand can strongly affect prices. This paper will discuss two *regime-switching, nonlinear stochastic models* of electricity prices that incorporate the effect of a time-varying demand on a market which can find itself in smooth or tight conditions in regard to demand dynamics.

II. Some Stylized Facts

In this paper, hourly price series will be considered. All data are obtained from the AESO Alberta (Canada) Electric System Operator web site [2]. Prices are expressed in Canadian dollars C$. The effect of demand on prices can be appreciated in Fig. 1, where one week of hourly prices is shown in Fig. 1(a). In Fig. 1(b) historical demand in MWh shows day/night periodicity, highest during daylight and with an extra prong after lunch time. In Fig. 1 electricity prices show their peculiar behavior in relation to demand. Prices tend to stay most of the time close to a baseline value, but sometimes rapidly rise and as much rapidly revert back to their original value, tracing the shape of a spike. Noticeably, prices do spike only during daylight. More precisely, sometimes they spike, sometimes they don't, but when they spike they do it only in coincidence with demand crests. Spikes appear occasionally but in well determined time windows. Sometimes, before reversion, high prices can *persist* for a while. Looking at longer stretches of data, demand shows even longer seasonality, weekly and yearly. Seasonality shows up in prices in at least two ways, with weekly and yearly changing baseline values and with seasonal changes of spiking frequency. All considered, price series show multiple reversion time scales, the time scale where spikes are involved being just the shortest. Spikes, seasonality and *complex mean reversion* are the most striking stylized facts of power prices. The phase-space of standard financial models is too simple to allow of this behavior.

The economic origin of the spikes is not completely clear. In an equilibrium approach, for a demand assumed inelastic, prices form at the clearing of the demand quantity q_d by a quantity-price supply curve $q_s(p)$ so that the relation $q_s(p) = q_d$ sets the equilibrium price $p_{eq} = q_s^{-1}(q_d)$. Power markets are auction markets, and this equilibrium approach makes sense. In a competitive environment, for a range of quantities, the supply side rationally proposes prices set at marginal costs for a selection of increasingly expensive technologies (the so-called 'power stack' [3]), which in aggregate take into account production *capacity constraints* [4], [5]. At high levels of demand, power markets can consequently become tight because of capacity limits. Power markets transfer energy through a *constraining power grid*, that at high demand can become *congested* [6]. In case of congestion, even in abundance of capacity, power markets can appear tight depending on demand. Last but not least, being power markets usually not really competitive, anticipated tight phases can allow of intentional or unintentional collusive behavior during these phases, a condition that makes prices even more volatile.

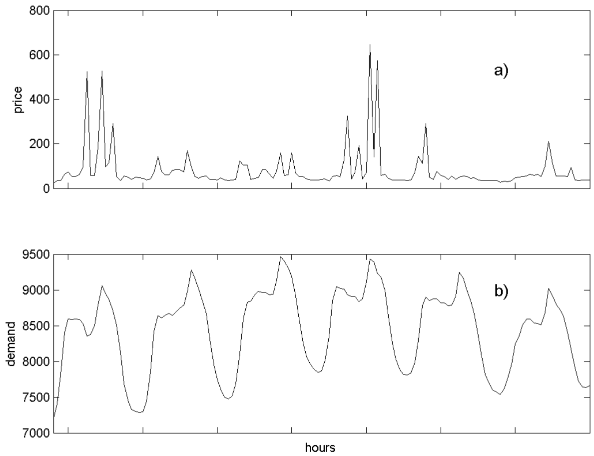

Fig. 1. Alberta power market: one week from Mon Jan-08-2007 to Sun Jan-14-2007, time in hours; a) system prices (SPs) in C$ - notice some spike persistence, b) demand in MWh.

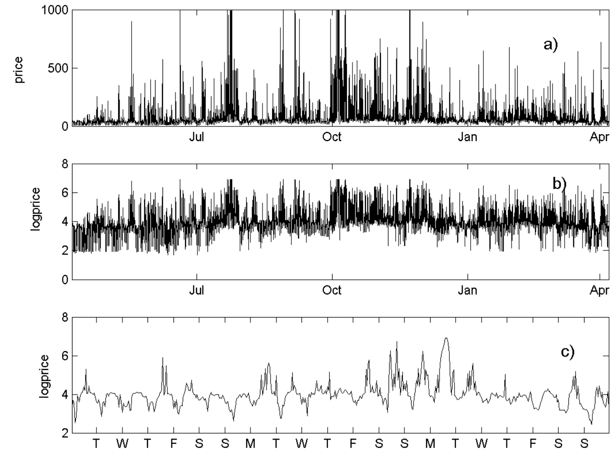

Fig. 2. Alberta power market prices; a) one year of hourly system prices p in c\$, from hour 1 of Apr-07-2006 to hour 24 of Apr-07-2007, time ticks in months; b) hourly system logprices x, same scale; c) detail of b): 3 weeks of logprices x from hour 1 of Aug-14 to hour 24 of Sep-3, time ticks in week days.

The main point here is that power markets behave as if they incorporate *demand threshold levels*, which when exceeded take the price system from one *regime* to - at least - a different one. The level of demand selects between a *normal regime* and a *potentially tight regime*. In the normal regime (demand off-peak hours) prices follow demand in a more or less linear way, in the potentially tight regime (demand peak hours) prices may react in a strongly nonlinear way - but not always they do it. In facts, above a certain demand threshold, in the potentially tight regime, prices can follow demand in either a linear or a nonlinear way, depending on whether capacity constraints or congestion set in. In Fig. 2(a), a 1 year time span of prices is shown, and in Fig. 2(b) the logarithm of the same series is displayed. Fig. 2(b) shows that, besides spikes, prices undergo also *antispikes*, maybe due to the impact of forward contracts. Spikes and antispikes have various heights, but three main logprice levels can be identified, i.e. a cap and a floor price level and a baseline price in between. The cap price corresponds to an institutional feature of the AESO market, where a maximum cap price cannot be exceeded. The floor price level is in principle zero (in contrast to other markets that can admit negative prices). Fig. 2(c) zooms in to show that, basically, spikes and antispikes seem to follow a similar dynamics, but in the reverse direction. Whereas spikes appear only during demand crests, antispikes appear only during demand troughs, and only occasionally. They are probably due to the delivery at low demand levels of forward contracts, struck long in advance at far lower prices. All considered, the overall dynamics seems then to switch occasionally from a normal regime to two other different and opposite spike regimes, a *potentially relaxed* and a *potentially tight* regime. In the log plot of Fig. 2(b) it is also evident a yearly baseline seasonality with its long reversion time mixing with spike short reversion time.

Modelling approaches that try to reproduce spiking and seasonality in path and distributional properties only, leaving aside considerations about the economical and technical origins of spiking, are called *top-down* approaches. Numerical interacting-agent approaches that accurately take into account technical and institutional considerations [7] are called *bottom-up* approaches. Intermediate *hybrid* approaches try to relate microeconomic features to stylized facts using small sets of equations.

III. Some Models

In continuous time, sophisticated models have been proposed for spike modelling taking a top-down approach. For example, Cartea and Figueroa [8] set up a first order mean reverting process with a time-dependent mean reverting level to which they attach a homogeneous Poisson process that triggers spikes. Geman and Roncoroni [9] design a similar process with an inhomogeneous Poisson process, using a non-smooth nonlinearity to have rising spikes to revert to the baseline level. More general Lévy processes can also be used for this purpose (consider for example the Benth approach [10]). All these models are based on jumps, i.e. price discontinuities. A typical example of a hybrid approach in continuous time not requiring jumps is an equilibrium model by Barlow [11] where spikes are obtained as equilibrium prices set by a nonlinear and increasingly expensive supply function q_s^{-1}.

In discrete time Misiorek, Trueck and Weron in Ref. [12] show that *regime-switching models*, in which the price dynamics can jump across different regimes that model in different ways baseline phases, spikes, or spiking phases, can easily include thresholds and demand effects. Consider the price stochastic dynamics $D(t)$ and a function $u = u(V, D)$ that can depend on an external *dynamic threshold variable* $V(t)$ associated with the model (observable or unobservable) and on past realized values of $D(t)$ itself. Typically, u is compared in value with a set $\{T_k|\ k \in K, T_1 < T_2 < \ldots < T_K\}$ of fixed thresholds T_k. Depending on the result of this comparison, the system dynamics $D(t)$ is then associated to one of the dynamic *regimes* of the set $\{R_k|\ k \in K+1\}$. If these regimes are represented by eXogenously-driven AutoRegressions (ARX) for $D(t)$, where the *exogenous driver* is $F(t)$, the model is called Threshold ARX (TARX) (for another example see Ref. [13]). If $u = u(D)$ depends only on $D(t)$ and doesn't depend on the external variable $V(t)$ the model is called *Self-Excited Threshold AutoRegressive eXogenously driven* (SETARX) model. A simple first-order $K = 1$ two-regimes example of a SETARX model is

$$\begin{cases} d_i = \phi_R\, d_{i-1} + f_{i-1} + \theta_R\, \epsilon_i, & u(d_{i-1}) \geq T, \quad R = R_1 \\ d_i = \phi_R\, d_{i-1} + f_{i-1} + \theta_R\, \epsilon_i, & u(d_{i-1}) < T, \quad R = R_2 \end{cases} \tag{1}$$

where $i = 1, \ldots, N$ are equally spaced discrete times, d_i belongs to the support of the system stochastic variable D, f_i belongs to the support of F, ϵ_i are i.i.d. draws from a distribution $P(\epsilon)$ and represent the *stochastic driver*, $R = R_1, R_2$ is the regime label, ϕ_R and θ_R stay constant within each regime R. The curly bracket is there to remind the multi-regime structure - notice that Eq. 1, despite its appearance, is a *single* SETAR(1)X equation. This first order difference dynamics is globally nonlinear, even though its dynamic equation is linear. A *second order* SETAR(2)X dynamics can be implemented either rising the difference order or coupling a second first order equation to Eq. 1.

Back to continuous time, switching first order continuous-time models can be called *regime-switching diffusions*. They can be used to model spikes, for example mixing an Ornstein-Uhlenbeck mean-reverting baseline diffusion with occasional outlier events (single point jumps) triggered by threshold crossing [14]. Calibration of this kind of jump models involves detection of jumps by *deseasonalization filtering* and some *spike filtering* procedure. It will be shown now that a suitably chosen SETAR(2)X discrete dynamics or its equivalent switching diffusion can sustain spikes without using jumps.

IV. The McKean Model

In continuous time, a model developed in mathematical neurobiology by H. P. McKean [15], discussed in depth in Ref. [16], can turn very useful in spike modelling and can easily be related to the SETARX and switching models frame. The McKean model is based on two coupled first order equations in the variables $X(t)$ and $Y(t)$, so that it is a second order process:

$$\epsilon \dot{x} = g_R(x;a) - y \tag{2a}$$
$$\dot{y} = x - \gamma_b\, y + b + f(t) + \sigma(d)\, \xi(t), \tag{2b}$$

In Eqs. 2, x and y belong to the support of X and Y, $\xi(t) = \frac{dW}{dt}$ (where $W(t)$ is a Wiener process) is the stochastic driver, the parameters $\epsilon > 0$, $\gamma_b > 0$, b, $\sigma(d) = \sqrt{2s}$, and a are constants that don't change at regime changes, $f(t)$ is the exogenous driver that will be deterministic and periodic, and g_R is defined as

$$g_R(x;a) = \begin{cases} -x, & -\infty < x \leq a/2, & R = R_1 \\ x - a, & a/2 < x < (1+a)/2, & R = R_2 \\ 1 - x, & (1+a)/2 \leq x < +\infty, & R = R_3. \end{cases} \tag{3}$$

Eqs. 2 can be easily discretized by stochastic Euler discretization. Eq. 3 shows how the first AR(1) component (no exogenous driver, then no X) of the discretized SETAR(2)X model of Eqs. 2 can switch among the three regimes $R = R_1, R_2, R_3$ as the observable function $u(x) = x$ is compared with the two thresholds $T_1 = a/2$ and $T_2 = (1+a)/2$, distinct but dependent on the single parameter a. Viewed in the context of SETARX literature, the McKean model has an interesting peculiarity that can be inferred considering the two *nullclines* of the system. These are the two curves in the $\{x(t), y(t)\}$ *phase-space* such that $\dot{x} = 0$ (x *nullcline*) and $\dot{y} = 0$ (y *nullcline*), when $\sigma = 0$:

$$y = g_R(x;a) \tag{4a}$$
$$y = (x + b + f(t))/\gamma_b \tag{4b}$$

Such curves can be seen for $f = 0$ in the phase-space displayed in Fig. 3, as a dashed and a dotted line respectively. The dashed piecewise-straight line is the x nullcline that results from Eq. 4a, the dotted straight upward sloping line is the y nullcline that results from Eq. 4b. For the moment,

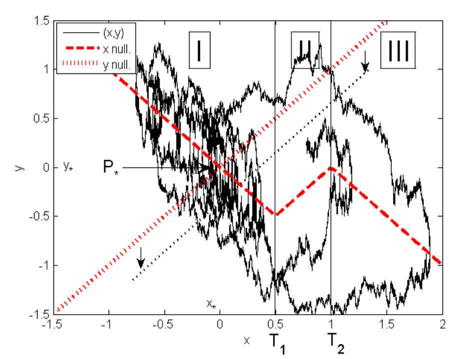

Fig. 3. McKean model phase-space for $f = 0$. Other parameters: $\epsilon = 0.3$, $s = 0.4$, $a = 1$, $b = 0$, $\gamma_b = 1$. Thick solid line: trajectory in phase-space. Dotted line: y nullcline (thinner dotted line pointed by two arrows: y nullcline shifted downward). Dashed line: x nullcline,. Regions are marked as I, II, III, thresholds as T_1 and T_2, the attractor as P_* with coordinates x_* and y_*.

take $f = 0$ and choose the parameters of Eqs. 2 as in Fig. 3: $a = 1$, $b = 0$, $\gamma_b = 1$. Consider Eq. 3 and look at the signs that $g_R(x;a)$ takes in front of x in the three regimes. It follows that the x nullcline has three linear pieces, which correspond to regimes $R = R_1, R_2, R_3$, due to the two thresholds T_1 and T_2.

Whereas the y nullcline is straight and has always a positive slope of value $1/\gamma_b = 1$ (Eq. 4b), the x nullcline is broken and shows, in a sequence from left to right, a R_1 negative slope value of -1, the T_1 threshold kink, a R_2 positive value of 1, the T_2 threshold kink, a R_3 negative value of -1. The three vertical bands across the phase-space of Fig. 3 will be called *regions* I, II, III and correspond to regimes R_1, R_2, R_3. When the condition expressed by taking Eq. 4a and Eq. 4b as a system $\dot{x} = \dot{y} = 0$ holds, the dynamics is quiescent - the time derivatives being 0 - so that the solution $P_* = \{x_*, y_*\}$ is an attractor for the dynamics. For the chosen parameters, P_* lies in region I and the attractor is *stable* because of the chosen local slopes of the two nullclines. Dynamic trajectories starting close to P_* will quickly end on P_* itself. Region I is *stable*. Yet, the three regions have different stability properties. Because of its slope, trajectories are also attracted by the x nullcline sector of region III, but this region is *metastable*, i.e. trajectories can remain for a while on this sector of the nullcline and then must leave. Region II is *unstable*. Trajectories tend to cross this region without stopping on it.

After activation of the stochastic driver ($\sigma \neq 0$), a typical system's phase-space trajectory $\{x(t), y(t)\}$ of the continuous-time McKean model for $f = 0$ is shown as a continuous thick line in Fig. 3. The system spends most of its time where the two nullclines cross each other, 'bubbling' around the point P_*. For two times, the noise $\xi(t)$ is able to kick the system from region I to region II, where it cannot stop, toward region III, where it can stay for a while then leaving back for region I again. During these two flights through region II, the process $x(t)$ describes two spikes of different heights, springing up and down from its baseline value x_*. *Spike persistence* appears when, during the return to region I through region II, a noise kick with the proper sign pushes the trajectory back to region III. The flights through region II can be eased if the nullcline y is taken closer to the second threshold T_2 (in Fig. 3, sliding the y nullcline downward parallel to itself in the direction of the arrows, by the use of a different value of the parameter b). In this case, it is easier for the stochastic driver $\xi(t)$ to have the chance to push trajectories from region I through threshold T_1 to the unstable region II, and start a spike. The process $x(t)$ can now be identified as a *logprice*

$$x(t) = \log p(t). \tag{5}$$

In Section II it was said that, at a first view, the level of demand selects between two regimes, a normal regime and a potentially tight regime. The first normal regime R_N can then be identified in the three-regimes McKean model with McKean regime R_1. The second potentially tight regime can be identified collectively with McKean regimes R_2 and R_3 as $R_{PT} = \{R_2, R_3\}$, since after the system enters region II from region I, it can *either* revert immediately to region I *or* continue to region III forming a spike. In this sense, the McKean model has only two regimes R_N and R_{PT} in accordance with the discussion of Section II, the threshold T_2 being assumed internal to regime R_{PT}.

V. Calibration: just a preliminary discussion

In the case of the discretized two-component McKean model of Eqs. 2, if an econometric calibration method like that in Ref. [17] has to be implemented, a first problem appears. The process $X(t)$ can be directly estimated on the logarithm of price data, but it might not be immediately clear on what data the auxiliary process $Y(t)$ could be estimated. It is then useful to rewrite Eqs. 2 as the second order single differential equation

$$\ddot{x} = (\frac{\partial}{\partial x} g_R(x;a) - \epsilon)\, \dot{x} + g_R(x;a) - (\gamma_b\, x + b) + f(t) - \sigma(d)\, \xi(t) \tag{6}$$

and notice that in turn this equation can be trivially represented by the system

$$\begin{aligned} \dot{x} &= z \quad &(7a)\\ \epsilon \dot{z} &= (\frac{\partial}{\partial x} g_R(x;a) - \epsilon)\, z \\ &\quad + g_R(x;a) - (\gamma_b\, x + b) + f(t) - \sigma(d)\, \xi(t) \quad &(7b) \end{aligned}$$

where z belongs to the support of a new auxiliary variable Z. Now

$$z = \dot{x} = \frac{\dot{p}}{p} \tag{8}$$

is clearly an *instantaneous logreturn*. After a discretization with time step Δt, Eq. 8 becomes the *logreturn* variable

$$z_i = (x_{i+1} - x_i)/\Delta t \tag{9}$$

and all data used for a *sample series* $\tilde{x}_i$ can be used to build the *auxiliary sample series* $\tilde{z}_i$. Eq. 7a and Eq. 7b become the components of a workable SETARX system where $g_R(x;a)$ is given by Eq. 3 and $\frac{\partial}{\partial x} g_R(x;a)$ is given by

$$\frac{\partial}{\partial x} g_R(x;a) = \begin{cases} -1, & -\infty < x \leq a/2, \quad R = R_1 \\ 1, & a/2 < x < (1+a)/2, \quad R = R_2 \\ -1, & (1+a)/2 \leq x < +\infty, \quad R = R_3. \end{cases} \tag{10}$$

The dynamics of $X(t)$ from Eqs. 7 is of course the same as that from Eqs. 2.

VI. Antispikes

Most papers on electricity price series neglect to model antispikes, being more concentrated on modelling spikes. The following SAS (Spike-AntiSpike) extension g_R^{SAS} of the function g_R allows to include antispikes as well:

$$g_R^{SAS}(x; C_L, C_R) = \begin{cases} -\alpha_L(x + C_L), & -\infty < x \leq -C_L, \\ & R = R_1 \\ \beta_L(x + C_L), & -C_L < x < -D_L = \frac{\beta_L}{\gamma_0+\beta_L}C_L, \\ & R = R_2 \\ -\gamma_0 x, & -D_L \leq x \leq D_R = \frac{\beta_R}{\gamma_0+\beta_R}C_R, \\ & R = R_3 \\ \beta_R(x - D_R), & D_R < x < C_R, \\ & R = R_4 \\ -\alpha_R(x - D_R), & C_R \leq x < +\infty \\ & R = R_5. \end{cases} \tag{11}$$

In Eq. 11 all parameters are positive and the system has five regimes, two of them unstable (regimes R_2 and R_4), two left thresholds $-C_L$, $-D_L$ and two right thresholds D_R, C_R. If α_L, β_L, γ_0, β_R, β_R are set equal to 1 in analogy with the simplifying choice of Eq. 3, $g_R^{SAS}(x; C_L, C_R)$ contains only two parameters (for example C_L and C_R), whereas $g_R(x; a)$ contains only one parameter a. In Fig. 4 the phase-space, the nullclines, the thresholds and the regions of the extended McKean model are shown. Again, coherently with the discussion of Section II, this five-regimes model can be interpreted as a three regimes model with a first *potentially relaxed antispike regime* where $R_{PRA} = \{R_1, R_2\}$, a second *normal regime* where $R_N = R_3$, and a third *potentially tight spike regime* where $R_{PTS} = \{R_4, R_5\}$. As for the McKean

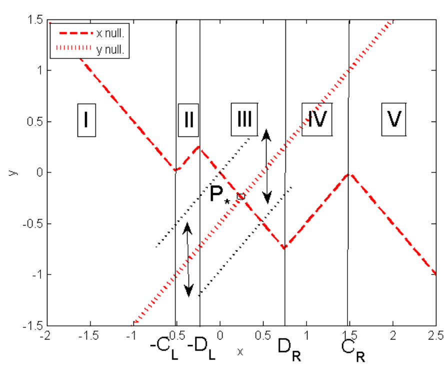

Fig. 4. Extended SAS McKean model phase-space for $f = 0$. Other parameters: $\alpha_L = \alpha_R = 1$, $\beta_L = \beta_R = 1$, $\gamma_0 = 1$, $C_L = 1/2$, $C_R = 3/2$, $b = -1/2$, $\gamma_b = 1$. Dotted line: y nullcline (thinner dotted lines pointed by two arrows: y nullcline shifted upward or downward). Dashed line: x nullcline. Regions are marked as I-V, thresholds as $-C_L$, $-D_L$, D_R, C_R, the attractor as P_*.

model of Eqs. 2 and Eq. 3, the extended SAS McKean model of Eqs. 2 (where g_R is replaced with g_R^{SAS}) and Eq. 11 can be rewritten and accommodated for SETARX econometric estimation - after due discretization - in the form of Eqs. 7, Eq. 11 and its derivative $\frac{\partial}{\partial x} g_R^{SAS}(x; C_L, C_R)$.

VII. McKean Models and Stochastically Resonating Spiking

In Section IV the exogenous driver $f(t)$ was considered for the moment equal to zero. When discussing the McKean dynamics of Eqs. 2 and Eq. 3, it was said that the flights through region II can be eased if the y nullcline of Eq. 4b is taken closer to the second threshold T_2 (shifting the y nullcline downward parallel to itself as in Fig. 3, by the use of a different value of the parameter b).

Another way to help noise to kick in is by use of the level of the sinusoidal driver term $f(t) = A\sin(\omega t)$ in Eq. 2b, which has the same effect as b but can slide the y nullcline upward and downward at fixed b in a periodic way. For a small A, P_* can be taken periodically close to lower threshold T_1 (but never letting it pass T_1), and periodically the system becomes more reactive to noise. This forcing can be interpreted as an effect of the electricity demand, and the first threshold T_1 as the soft border of an aggregate and potentially tightened market condition of the power system in respect to a high demand, due to either capacity constraints or grid congestions, or to both. When noise is able to kick the system into region II, a spike is fired as the trajectory tries to reach region III. In this way, spike activity is mostly probable only during demand crests and it is suppressed in probability during demand troughs. The frequency of the exogenous driver f can be set accordingly to the periodicity of the market, being the 24 hours seasonality the most obvious for hourly data. If an exogenous driver $f(t) = A_0\sin(\omega_0 t) + A_1\sin(\omega_1 t)$ contains extra frequencies, time scales beyond the single-spike daylight scale can be included. When needed, spiking activity can be enhanced in specific seasons (e.g. in winter, when nordic countries like Alberta use electric heating) while retaining the same price baseline level of other seasons. During peak season the threshold T_1 will find itself closer to the demand peaks and spikes will be more frequent - but will retain their low season baseline and peak-to-base structure. This combination of *forcing, nonlinearity and noise* was exploited (for a different model) as a spiking mechanism for electricity market prices in a model introduced in Ref. [18] and developed in Ref. [19] and Ref. [20], under the name of *stochastically resonating spiking* (SRS). In Ref. [18] more information can be found about the role of ϵ in Eqs. 2 (here tuned to the *soft ϵ regime*) and in Ref. [19] a discussion can be found about the possible uses of dynamic system bifurcation theory and Hopf critical points in seasonally and irregularly peaking commodity markets.

An example of the SRS mechanism can be given by simulating the continuous-time extended SAS McKean model of Eqs. 2 and Eq. 11 with a nonzero $f(t) = A\sin(\omega t)$. The y nullcline and P_* are periodically forced upward and downward respect to the x nullcline in the corridor sketched in Fig. 4 limited by the two thinner dotted lines. A typical trajectory of a forced extended SAS McKean dynamics is shown in Fig. 5. Again, process $x(t)$ is identified with the logprice, so that the price is $\exp(x)$. Fig. 5(a) shows the phase-space, the two nullclines and a representative trajectory

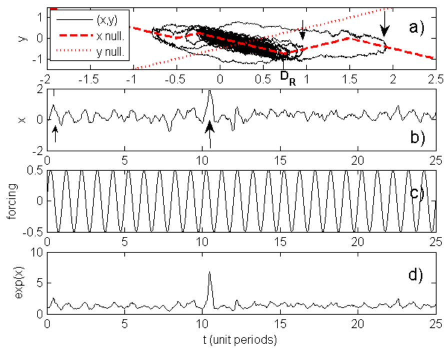

Fig. 5. Extended McKean model for $f \neq 0$. Parameters: $\epsilon = 0.3$, $s = 0.1$, $\alpha_L = \alpha_R = 1$, $\beta_L = \beta_R = 1$, $\gamma_0 = 1$, $C_L = 1/2$, $C_R = 3/2$, $b = -1/2$, $\gamma_b = 1$, $A = 0.5$, $\omega = \pi/2$. a) phase-space and nullclines, the threshold D_R is indicated under the x axis, a spike is indicated by a big arrow, a would-be spike is indicated by a small arrow, b) logprice $x(t)$ dynamics, the same spike is indicated by a big arrow, the same would-be spike is indicated by a small arrow, c) forcing $f(t) = A\sin(\omega t)$, d) price $\exp x(t)$ dynamics.

$\{x(t), y(t)\}$. The system spends most of its time in the normal region III. Fig. 5(b) shows the obtained logprice trajectory, to be compared visually with the market data in Fig. 2(c). Fig. 5(c) shows the demand (i.e. forcing) dynamics, with hourly periodicity. When demand pushes the system close to the right threshold D_R of region IV (i.e. of the potentially tight spike regime), it can happen (but not necessarily) that a spike is fired, This happens once in Fig. 5(a), where the spike is indicated by a big arrow. The parameter ϵ controls how much the system is repelled from the unstable regions II and IV. The choice of $\epsilon = 0.3$ allows the system to explore partially the unstable regions, specifically region IV. This can be seen in Fig. 5(a), locating the value of $D_R = 3/4$ on the x axis and observing that even though this threshold is crossed from left to right, in some cases the trajectory is forced back to region III very quickly without ever reaching region V. In Fig. 5(a) and in the $x(t)$ graph of Fig. 5(b), this bounce appears as a *would-be spike* with a very small height, indicated by a small arrow. In other continuous-time models the presence of would-be spikes is obtained by the use of Lévy dynamics. When demand pushes the system close to the left threshold $-D_L$ of the potentially relaxed spike regime R_{PRS}, it can happen (but not necessarily) that an antispike is fired, and this happens twice or three times in Fig. 5(a). Differently from what happens in this example, the generic nullcline structure of Eq. 11 is not symmetric, so that for example a small antispike can be made more probable than a large spike. This flexibility is intended to accommodate in the model many market phenomenologies. Fig. 5(d) shows the price process, where antispikes are much less evident, which could be the reason why antispikes are not very often studied, even though they contribute heavily to the dynamics and to the quality of its calibration.

VIII. Conclusions

At this point it could be clear that the strength of the forced McKean models is that they can accommodate multiple mean reversions by the same SRS mechanism. They always include at least one or two (when antispikes are considered) fast reversion time scales. This feature is due to their topological structure, i.e. their uncommon spiking regime TARX structure - unstable plus metastable regions in due sequence, whereas in usual TARX models only stable structures are used. In Section II it was mentioned that seasonality shows up in data in two ways, seasonal changes in spiking frequency, and seasonal changes of baseline price values. By the addition of terms to the exogenous driver $f(t)$, extra spiking seasonality can be included. Moreover, McKean models can be modified to accommodate seasonal baseline changes, and this will be shown in further work.

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