A Phasor Estimation Algorithm for a Decaying AC Component

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Abstract— **Conventional phasor estimation methods can causes error if the envelope of the signal is changed according to time. In this paper, a modified dynamic phasor estimation method was proposed to overcome the disadvantages of conventional phasor estimation methods. To evaluate the performance of the proposed method, a fault location algorithm which uses the proposed phasor estimation method was tested over varying fault distances and fault resistances. The fault location result using the proposed phasor estimation shows a very small error compared to fault location methods using conventional phasor estimation.**

Keywords- Modified dynamic phasor, generator transient, phasor estimation

I. INTRODUCTION

Definition of phasor is an expressive method of steady state in which magnitude and angle are fixed. Since voltage and current signal of a power system are expressed using a phasor, accuracy of an algorithm using a phasor depends on accuracy of phasor estimation. Discrete Fourier, Walsh, Harr, Least Square or Kalman filtering have been used for phasor estimation [1-7].

The power system is composed of many generators having a transient feature that show a different response according to the time of sub-transient, transient and synchronous periods. Accordingly, the dynamic characteristic of a generator can be shown such that the magnitude of current is changed by time if a transient occurs at a point near a generator. If a steady state analysis is used during such a period, it doesn't meet the definition and it causes an error. Therefore, a new estimation method that is capable of detecting changes in magnitude according to time is required. As a phasor estimation technique during such a transient period, dynamic phasor estimation techniques were introduced[8-12]. These methods use complex Taylor Fourier series to estimate the phasor, but they don't consider DC offset, and are used at the division for stability analysis or power system control. For another method, phasor estimation using the Prony techniques were introduced[13-14]. In this method, an accurate result is assured if the assumed signal coincides with the input signal; however, an equation of higher degree is required for the higher order input signal assumption. It has weak even when the signal includes little noise.

In this paper, the modified dynamic phasor estimation method was proposed to estimate a phasor that changes magnitude according to time during the transient period. A Taylor series was used for expression of exponentially decaying fundamental component. The phasor was estimated based on the Least Square technique. The DC offset of input signal was removed by using additional method before applying the modified dynamic phasor estimation.

To verify the proposed method, phasor estimation result of a test signal and a fault location algorithm using the proposed phasor estimation method were shown. Errors in fault location result using the proposed phasor estimation show a very small error compare to errors of fault location using conventional phasor estimation.

II. MODIFIED DYNAMIC PHASOR ESTIMATION METHOD

A. *Short circuit current of a generator in transient period*

When a three- phase short circuit fault occurs at the generator terminal fault current can be expressed with generator terminal voltage and source impedance in (1).

$$i_a = \sqrt{2}|E(0)|\left[\begin{array}{l}\dfrac{1}{X_d}+\left(\dfrac{1}{X_d^{'}}-\dfrac{1}{X_d}\right)e^{-t/T_d^{'}}\\ \quad +\left(\dfrac{1}{X_d^{''}}-\dfrac{1}{X_d^{'}}\right)e^{-t/T_d^{''}}\end{array}\right]\sin(\omega_0 t+\delta) \tag{1}$$

Where,

$E(0)$: Open circuit voltage
X_d : Synchronous reactance
$X_d^{'}$: d - axis transient reactance
$X_d^{''}$: d - axis sub - transient reactance
$T_d^{'}$: d - axis transient time constant
$T_d^{''}$: d - axis sub - transient time constant

Figure. 1 shows the decaying of magnitude of a short circuit current in transient and sub-transient period. It indicates that the source impedance of a generator changes according to time while the current changes exponentially. Since the conventional phasor analysis is a steady state analysis, it is not appropriate to estimate decaying signal therefore a technique to accurately estimate phasor that changes envelope according to time is required.

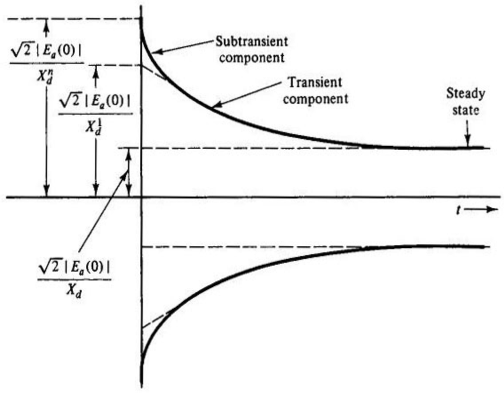

Figure 1. Three phase short circuit current of a generator in transient period

B. The modified dynamic phasor estimation algorithm

The dynamic phasor technique has the disadvantage of being incapable of responding to DC offset. The Prony method has a weakness in that higher order degree equation is required to obtain accurate estimation. Error increases if an unexpected factor is included in assumption of input signal. Therefore, in this paper, the modified dynamic phasor estimation technique is suggested in order to overcome the disadvantages of these methods.

Envelope of a phasor was estimated by assuming an input signal inclusive of a Taylor series of fundamental frequency component and high frequency components. To remove DC-offset, an additional removal method was used.

$$y(t) = \left(A_0 + A_1 e^{-t/\tau_{ac1}} + A_2 e^{-t/\tau_{ac2}}\right)\sin(wt+\theta) + \sum_{m=2}^{M} A_m \sin(mwt+\theta_m) \tag{2}$$

If input signal is assumed by the sum of fundamental frequency component that changes magnitude according to time and high frequency components the signal can be expressed as (2). Equation (2) can be rearranged as (3) using a trigonometric function.

$$\begin{aligned} y(t) &= \left(A_0 + A_1 e^{-t/\tau_{ac1}} + A_2 e^{-t/\tau_{ac2}}\right)\sin\theta\cos wt \\ &+ \left(A_0 + A_1 e^{-t/\tau_{ac1}} + A_2 e^{-t/\tau_{ac2}}\right)\cos\theta\sin wt \\ &+ A_m \sum_{m=2}^{M} \left(\sin\theta_m \cos(mwt) + \cos\theta_m \sin(mwt)\right) \end{aligned} \tag{3}$$

The section of fundamental components that has a coefficient of $\cos wt$ is defined as $p(t)$ in (3). $p(t)$ can be expressed by using Taylor series, the following (4) is obtained. Equation (4) is organized for time, (5) is presented.

$$\begin{aligned} p(t) &= A_0\cos\theta + \left(A_1 \frac{t}{\tau_{ac1}}\cos\theta + A_1 \frac{t^2}{\tau_{ac1}^2}\cos\theta\right) \\ &+ \left(A_2 \frac{t}{\tau_{ac2}}\cos\theta + A_2 \frac{t^2}{\tau_{ac2}^2}\cos\theta\right) \end{aligned} \tag{4}$$

$$p(t) = p_0 + p_1 t + p_2 t^2 \tag{5}$$

Where,

$$p_0 = (A_0 + A_1 + A_2)\cos\theta$$

$$p_1 = \left(A_1 \frac{1}{\tau_{ac1}} + A_2 \frac{1}{\tau_{ac2}}\right)\cos\theta$$

$$p_2 = \left(A_1 \frac{1}{\tau_{ac1}^2} + A_2 \frac{1}{\tau_{ac2}^2}\right)\cos\theta$$

The section of fundamental components that has a coefficient of $\sin wt$ is defined as $r(t)$. Equation (7) can be obtained from (6) in the same way.

$$\begin{aligned} r(t) &= A_0\sin\theta + \left(A_1 \frac{t}{\tau_{ac1}}\sin\theta + A_1 \frac{t^2}{\tau_{ac1}^2}\sin\theta\right) \\ &+ \left(A_2 \frac{t}{\tau_{ac2}}\sin\theta + A_2 \frac{t^2}{\tau_{ac2}^2}\sin\theta\right) \end{aligned} \tag{6}$$

$$r(t) = r_0 + r_1 t + r_2 t^2 \tag{7}$$

Where,

$$r_0 = (A_0 + A_1 + A_2)\sin\theta$$

$$r_1 = \left(A_1 \frac{1}{\tau_{ac1}} + A_2 \frac{1}{\tau_{ac2}}\right)\sin\theta$$

$$r_2 = \left(A_1 \frac{1}{\tau_{ac1}^2} + A_2 \frac{1}{\tau_{ac2}^2}\right)\sin\theta$$

As shown in (4) and (6), input signal has two different time constants, and the result is organized into the common term of time t as (5) and (7). It seems that the Taylor series of the equation has one time constant but the terms of $p(t)$, $r(t)$ consist of the steady state value and transient sate value. Input signal (1) can be expressed as (8) by using (5) and (7); then, The the point of view of the phasor, the real part and the imaginary part of the fundamental component can be expressed as (9).

$$y(t) = p(t)\cos wt + r(t)\sin wt + \sum_{m=2}^{M} A_m\left(\sin\theta_m \cos(mwt) + \cos\theta_m \sin(mwt)\right) \tag{8}$$

$$Y_{real} = r(t) = r_0 \qquad Y_{imag} = p(t) = p_0 \tag{9}$$

The modified dynamic phasor can estimate the phasor including the term of one steady state and two decaying fundamental components. The input signal which consists of a decaying fundamental component and a harmonic component up to the fifth order can be expressed as (10).

$$y(t) = (r_0 + r_1 t + r_2 t^2)\cos wt + (p_0 + p_1 t + p_2 t^2)\sin wt + \sum_{m=2}^{5} A_m\left(\sin\theta_m \cos(mwt) + \cos\theta_m \sin(mwt)\right) \tag{10}$$

To solve (10), equations of the number of unknown variables are required. In (11), matrix A is input signal, matrix D is known variables and matrix E is unknown variables.

$$A[14\times 1] = D[14\times 14]E[14\times 1] \tag{11}$$

Where,

$$A = [y(t_0)\; y(t_1) y(t_2)\; y(t_3) \ldots . y(t_{13})]^T$$

$$D = \begin{bmatrix} \sin(wt_0) & t_0\sin(wt_0) & t_0^2\sin(wt_0) & \cos(wt_0) & t_0\cos(wt_0) & t_0^2\cos(wt_0) & \sin(2wt_0) & \cos(2wt_0) & \cdots & \sin(5wt_0) & \cos(5wt_0) \\ \sin(wt_1) & t_1\sin(wt_1) & t_1^2\sin(wt_1) & \cos(wt_1) & t_1\cos(wt_1) & t_1^2\cos(wt_1) & \sin(2wt_1) & \cos(2wt_1) & \cdots & \sin(5wt_1) & \cos(5wt_1) \\ \sin(wt_2) & t_2\sin(wt_2) & t_2^2\sin(wt_2) & \cos(wt_2) & t_2\cos(wt_2) & t_2^2\cos(wt_2) & \sin(2wt_2) & \cos(2wt_2) & \cdots & \sin(5wt_2) & \cos(5wt_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \sin(wt_n) & t_n\sin(wt_n) & t_n^2\sin(wt_n) & \cos(wt_n) & t_n\cos(wt_n) & t_n^2\cos(wt_n) & \sin(2wt_n) & \cos(2wt_n) & \cdots & \sin(5wt_n) & \cos(5wt_n) \end{bmatrix}$$

$$E = [p_0\; p_1\; p_2\; r_0\; r_1\; r_2\; A_2\cos\theta_2\; A_2\sin\theta_2 \;\ldots .\; A_5\cos\theta_5 A_5\sin\theta_5]^T$$

The Least Square based algorithm shows a more stable result when more equations are used over the number of the unknowns. Because matrix D is not a square matrix unknowns can be acquired as in (12).

$$[E] = \left[D^T D\right]^{-1}[D]^T[A] \tag{12}$$

C. Estimation result of the proposedalgorithm for a generated test signal

$$y(t) = 10e^{-\frac{t}{0.05}} + \left(10e^{-\frac{t}{1.0}} + 5\right)\sin wt + 3\sin(2wt) + 2\sin(3wt) + \sin(4wt) + 0.5\sin(5wt) \tag{13}$$

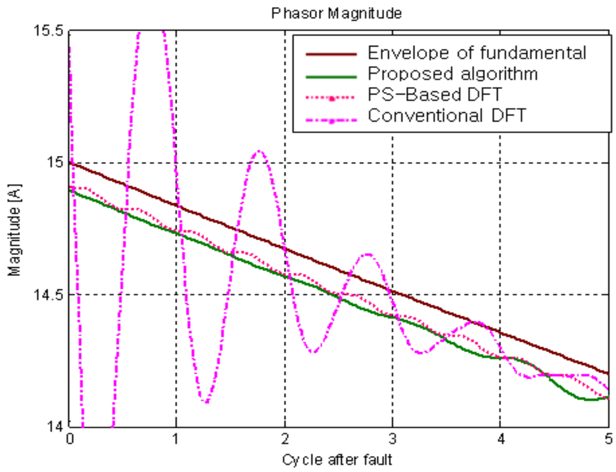

Figure 2. Phasor estimation result in case of a test signal

Equation (13) is a generated test input signal. Figure 2 shows the estimated magnitude of several methods. The proposed phasor estimation method shows the most stable result compare to conventional DFT or PS based DFT. PS based DFT was presented in [15].

III. FAULT LOCATION ALGORITHM

When a line-to-ground fault occurs in Figure 3, the voltage equation can be expressed as (14) by using the negative sequence current distribution factor. Equation (14) can be expressed as (15). The per unit distance p and fault resistance R_f can be estimated by solving the two equations obtained by separating the complex equation into the real and the imaginary part.

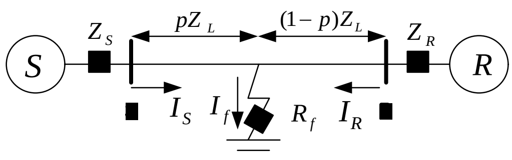

Figure 3. Line-to-ground fault

$$V_{Sa} = p[Z_{L1}I_{Sa} + (Z_{L0} - Z_{L1})I_{S0}] + \frac{3R_f I_{S2}}{CDF_2} \tag{14}$$

Where,

$$CDF_2 = \frac{I_{S2}}{I_{f2}} = \frac{1}{\left(1+\dfrac{I_{R2}}{I_{S2}}\right)} = \frac{1}{\left(1+\dfrac{pZ_{L2}+Z_{S2}}{(1-p)Z_{L2}+Z_{R2}}\right)}$$

$$a_1 p^2 + a_2 p + a_3 + a_4 R_f = 0 \tag{15}$$

Figure 4 is the negative sequence network in case of a line-to-ground fault. Because there is no negative sequence voltage behind the local relaying point at S in the negative sequence network, the negative sequence source impedance can be estimated by using after-fault data as in (16).

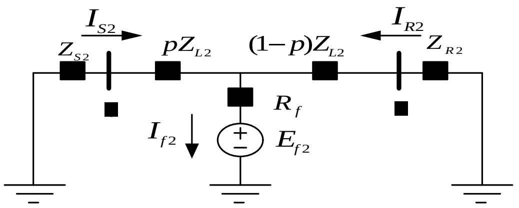

Figure 4. Negative sequence circuit

$$Z_{S2} = -\frac{V_{S2(Post-fault)}}{I_{S2(Post-fault)}} \tag{16}$$

In a same manner, when a double line-to-ground fault occurs, the line-to-line voltage at the relaying point is given like (17) and it can be modified like (18). The estimated negative sequence source impedance is used for the CDF_2 .

$$V_{Sbc} = pZ_{L1}I_{Sbc} + R_f I_f \tag{17}$$

$$V_{Sbc} = pZ_{L1}I_{Sbc} + R_f \frac{1}{CDF_2}\left[\left(\alpha^2-\alpha\right)I_{S1} + \left(\alpha-\alpha^2\right)I_{S2}\right] \tag{18}$$

Where, $\alpha = e^{-j2\pi/3}$

Equation (18) is valid for a line-to-line fault also. Equation (18) can be expressed like (19) by using (20) to avoid using the positive sequence component.

$$V_{Sbc} = pZ_{L1}I_{Sbc} + j\sqrt{3}\frac{R_f I_{S2}}{CDF_2} \tag{19}$$

$$I_{f2} = -I_{f1}, \quad I_{f2} = \frac{1}{3}\left(\alpha^2-\alpha\right)I_f = -j\frac{I_f}{\sqrt{3}} \tag{20}$$

IV. CASE STUDY

To evaluate the performance of the proposed method, fault signals are simulated by the PSCAD/EMTDC on a 154kV, 25km single circuit transmission line system as in Figure 5. The local end is composed of four synchronous generators.

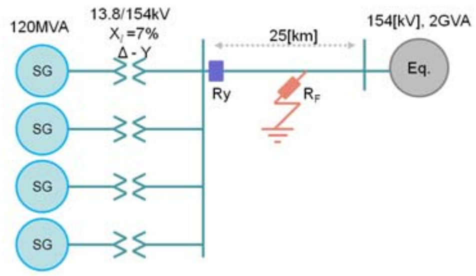

Figure 5. Model system

The proposed algorithm was tested with varying fault distances, fault resistances and source impedances. The sampling frequency of the algorithm was 1920(Hz). A 2nd-order Butterworth lowpass filter whose stop-band cutoff frequency is 960Hz was used for preventing aliasing error. The error of fault location is expressed as a percentage of the total line length in (21).

$$\%Error = \frac{|estimated\ \ location\ - \ actual\ \ location|}{total\ \ length} \times 100 \tag{21}$$

To show the accuracy of the proposed algorithm, errors of fault location algorithm which use both the modified dynamic phasor estimation and local source impedance estimation were compared to the errors of the algorithm which used both conventional phasor estimation method and fixed local source impedance.

Figure 6 and 7 show the location errors with the varying fault distances and fault resistances in case of line-to-ground fault. Conventional phasor estimation and fixed source impedance were used for fault location algorithm. Figure 8 and 9 show the location errors with same faults in case of using proposed phasor estimation algorithm and estimated source impedance.

A. *A phase line-to-ground fault(Conventional)*

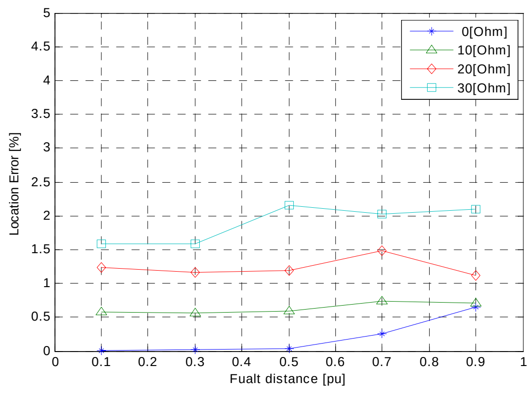

Figure 6. Fault location error (Conventional phasor estimation, fixed source impedance used, fault inception angle: 0°)

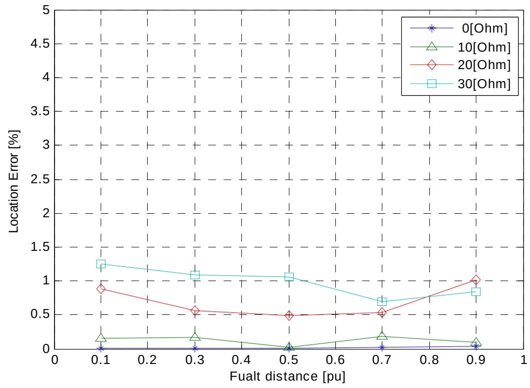

Figure 7. Fault location error (conventional phasor estimation, fixed source impedance used, fault inception angle: 90°)

B. A phase line-to-ground fault(Proposed)

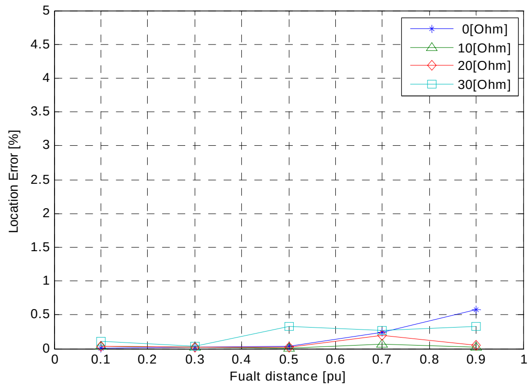

Figure 8. Fault location error (Proposed phasor estimation, source impedance estimation, fault inception angle: 0°)

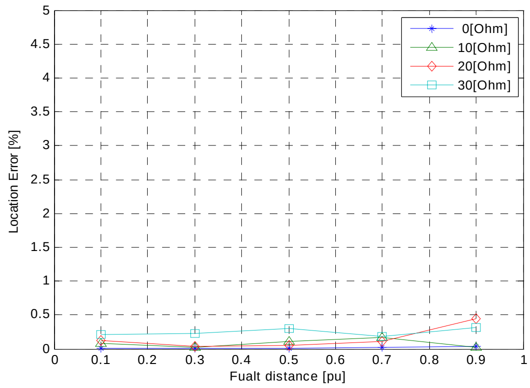

Figure 9. Fault location error (Proposed phasor estimation, source impedance estimation, fault inception angle: 90°)

Fault location results using the proposed phasor estimation shows a very small error compared to the fault location method using conventional phasor estimation. The maximum error of the proposed method is less than 0.5% in all cases. On the other hand, errors of the compared algorithm are bigger.

C. BC phase line-to-line fault(Conventional)

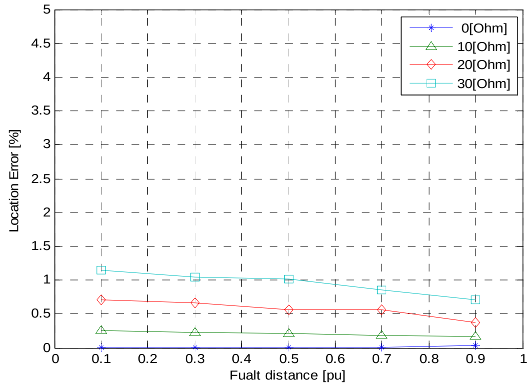

Figure 10. Fault location error (Conventional phasor estimation, fixed source impedance used, fault inception angle: 0°)

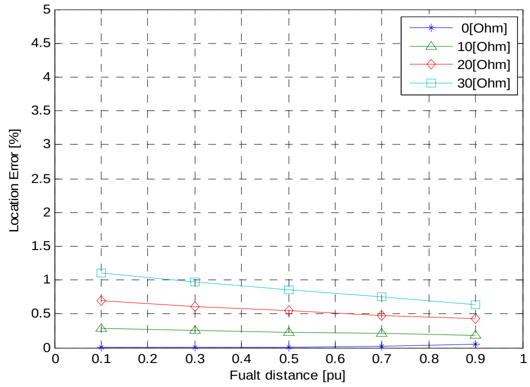

Figure 11. Fault location error (Conventional phasor estimation, fixed source impedance used, fault inception angle: 90°)

D. BC phase line-to-line fault (Proposed)

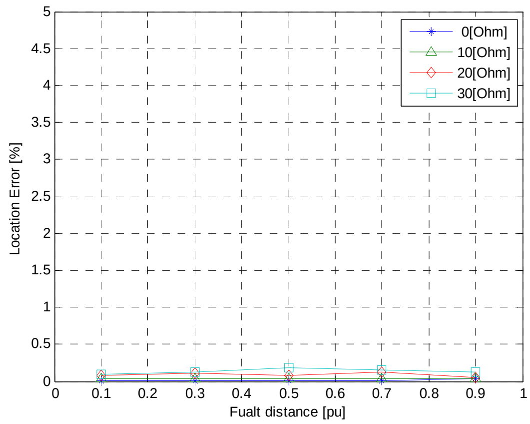

Figure 12. Fault location error (Proposed phasor estimation, source impedance estimation, fault inception angle: 0°)

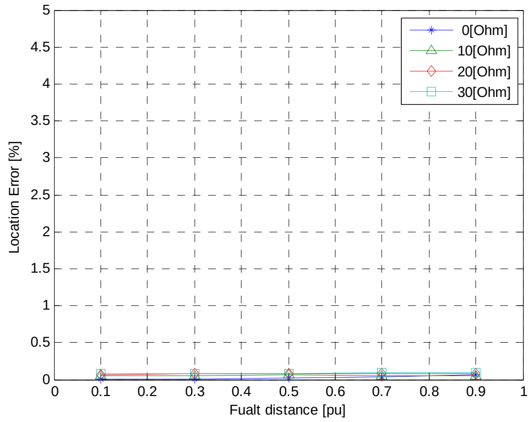

Figure 13. Fault location error (Proposed phasor estimation, source impedance estimation, fault inception angle: 90˚)

Figure 10~13 show the location errors in case of line-to-ground fault. As figures 12 and 13 shown, errors of fault location algorithm which uses the proposed phasor estimation and the estimated source impedance is smaller than those of the conventional phasor estimation and fixed source impedance. The maximum error is less than 0.2(%) .

V. CONCLUSION

The conventional phasor method can cause estimation error if the envelope of the signal is changed according to time. In this paper, a modified dynamic phasor estimation method was proposed to overcome weaknesses of the conventional phasor estimation methods. To evaluate the performance of the proposed method, fault location algorithm which uses the proposed phasor estimation method was tested over varying fault distances and fault resistances. Fault location using the proposed phasor estimation shows more accurate results compare to fault location methods using conventional phasor estimation.

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