Generation Bidding Strategy based on Game Theory

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***Abstract*—This paper presents a game theory application for analyzing power transaction in a deregulated energy market place such as poolco, where participants, especially, generating entities, maximize their net profit through optimal bidding strategies (i.e. bidding prices and bidding generations). In this paper by using game theory to simulate the decision making process for defining offered prices in a deregulated environment. The outcome of this study is to discourage unfair coalitions. A modified IEEE 30 bus system is used as a deregulated power pool to illustrate the main features of the proposed method.**

Keywords- Power systemr operation Deregulation,game theory,poolco model,spot price.

I. INTRODUCTION

Restructuring of electricity supply industry, worldwide, has brought the market competitiveness to the forefront, but the emergent electricity markets have a variety of new issues such as oligopolistic nature of the market, supplier's strategic bidding, market power abuses, price-demand elasticity and so on. Theoretically, in a perfectly competitive market, suppliers should bid at, or very close to, their marginal production cost to maximize payoff. Also producers, who are small enough to affect market prices with their bids, are price takers, and their optimal strategy is to bid at the marginal cost of production [1]. However, the electricity markets are oligopolistic in practice, and power suppliers may seek to increase their profit by bidding a price higher than marginal production cost. Knowing their own costs, technical constraints and their anticipation of rival and market behavior, suppliers face the problem of constructing the best optimal bid. This is known as a strategic bidding problem.

In this paper a methodology based on the non cooperative Game Theory [2–4] is used to analyze the economic behavior of the generating companies. The pool is modeled as a strategic game in which participants *play* against each other in to maximize their own benefits. The *strategy* that participants follow is the bid for pricing transactions. From the point of view of pool coordinators, the methodology presented in this paper is briefly given as:

- Identify players of the game and their possible strategies
- Identify possible coalitions among participants
- Compute transactions and economic benefits associated with any coalition
- Identify those coalitions which are likely to be formed
- Encourage those coalitions that would maximize pool benefits

Network constraints are considered in the formulation. A modified IEEE 30 bus system is used to demonstrate the proposed method.

II. GAME THEORY

The Game Theory can be defined as the study of mathematical models of conflict and cooperation between decision-makers. The Game Theory is a mathematical technique that analyzes situations in which two or more individuals make decisions that influence one another's welfare. In theory, a game refers to any social situation, which involves two or more players. Two basic hypotheses exist that are made about the players: they are rational and intelligent. Each of these adjectives is employed in a technical sense. A player is rational if he makes consistent decisions with the achievement of his own objectives. It is supposed that the aim of each player is to maximize the expected value of its own payment, which is measured in some scale of utility. A player is intelligent if he knows everything that is relative to the game and can make inferences concerning the situations, which can take place.

In Game theory two different approaches, one is the strategic or non cooperative approach. This requires a very detailed specification of the rules of the game, so that the strategies available to the players could be known in detail. The objective is to find an adequate group of strategies of equilibrium, which will be called the solution of the game. What is best for a player depends on what the other players think to do and this in turn depends on what they think the first player will do. These games are called strictly competitive or of zero-sum because any player's gain is always exactly balanced with a loss corresponding to the other player. The solution of games of non zero-sum, those in which the gain of a player is not the same as the loss of others, was first formulated by John Nash [5].The application of the games with complete information is presented in [6]. The application of the games with incomplete information is presented in [7]. The other approach is the coalition or cooperative, which adopts a less rigid attitude [8-13] .The analysis, is made resorting to the criterion max–min (or characteristic function) to make decisions based on the use of pure strategies.

III. WHOLE SALE COMPETITIVE SPOT MARKET

A. *The Mathematical Model*

The problem of the active power ED of a thermal electric power system can be set a non-linear optimization problem, subject to the generation-load balance, the technological characteristic of the generating units, and the capacity of the transmission system restrictions [14, 15]:

Minimize $\left\{ \sum_{i=1}^{M} C_i\left(P_{Gi}\right) \right\}$

Subjected to: $\sum_{i=1}^{M} P_{Gi} = P_L + \sum_{j=1}^{N} P_{Dj}$

$P_{Gi}^{Min} \le P_{Gi} \le P_{Gi}^{Max}$; *i = 1……TL*

$P_k \le P_K^{Min}$;*k= 1…..TL* (1)

Where

$C_i(P_{Gi})$ is production cost of the unit *i*

P_{Gi} is active power output of the unit *i*

P_{Dj} is active power load at bus *j*

M is number of generating units

N is number of system buses

P_k, P_k^{Max} are active power flow and its limit on line *k*;

P_{Gi}^{min} P_{Gi}^{max} are active power limits of the unit *i*

P_L is transmission losses

TL is number of transmission line.

The Lagrange function is:

$$\mathrm{L}\left(P_{G1\ldots\ldots\ldots\ldots}P_{GM}\right)$$

$$= \sum_{i=1}^{M} C_i\left(P_{Gi}\right) + \lambda\left[\sum_{j=1}^{N} P_{Dj} + P_L - \sum_{i=1}^{M} P_{Gi}\right]$$

$$+ \sum_{i=1}^{M}\left[\mu_i^{Min}\left(P_{Gi}^{Min} - P_{Gi}\right) + \mu_i^{Max}\left(P_{Gi} - P_{Gi}^{Max}\right)\right]$$

$$+ \sum_{k=1}^{TL}\left[\sigma_k\left(P_k - P_k^{Max}\right)\right] \quad (2)$$

Where

λ is Lagrange multiplier (generation-load balance)

μ_i^{Min} μ_i^{Max} are Lagrange multipliers (technological characteristics of the generating units)

σ_i is Lagrange multipliers (capacity of the transmission system).

The marginal costs of the thermal units are affected by the spatial and temporal quantification of the transmission losses, via the penalty factors of the corresponding generation bus, which are defined in the classic ED of active power by:

$$f_{pi} = \left[1 - \frac{\partial P_L}{\partial P_{Gi}}\right]^{-1} \quad (3)$$

The necessary conditions of the first order of Karush–Khun–Tucker [16] are:

for *i*=1... *M*

$$f_{pi}\frac{dC_i}{dP_{Gi}} = \lambda \quad \text{if} \quad P_{Gi}^{Min} \le P_{Gi} \le P_{Gi}^{Max}$$

$$f_{pi}\frac{dC_i}{dP_{Gi}} \ge \lambda \ \text{ if } \ P_{Gi} = P_{Gi}^{Min}$$

$$f_{Pi}\frac{dC_i}{d_{Gi}} \le \lambda \text{ , if } \ P_{Gi} = P_{Gi}^{Max}$$

$$\sum_{i=1}^{M} P_{Gi} = P_L + \sum_{j=1}^{N} P_{Dj}$$

$$P_k \le P_k^{\max} \text{ ;}k\text{= 1………………..}TL \quad (4)$$

The inclusion of active restrictions (LIN) into Eq. (4) is given by:

for i=l,…, *M*

$$P_{Gi}^{Min} \le P_{Gi} \le P_{Gi}^{Max}$$

$$f_{pi}\left[\frac{dC_i}{dP_{Gi}} + \sum_{k\in LIN}\sigma_k\frac{\partial P_k}{\partial P_{Gi}}\right] \ge \lambda$$

or, if $P_{Gi} = P_{Gi}^{Max}$

$$f_{pi}\left[\frac{dC_i}{dP_{Gi}} + \sum_{k\in LIN}\sigma_k\frac{\partial P_k}{\partial P_{Gi}}\right] \le \lambda$$

$$\sum_{i=1}^{M} P_{Gi} = P_L + \sum_{j=1}^{N} P_{Dj}$$

$$P_k \le P_k^{Max}; k \notin LIN$$

$$P_k = P_k^{Max}; k \in LIN$$

$$\sigma_k \ge 0; k \in LIN \quad (5)$$

If the restrictions on the transmission capacity are not active, in the optimum, all thermal units operate at market price (λ of the system) within its technological limits. For the ones which generate their minimum (or maximum) power, they operate, respectively, at *i*-bus price:

$$\rho_i = f_{pi} \frac{dC_i}{dP_{Gi}} \qquad (6)$$

If the restrictions on the transmission capacity are active, in the optimum, all thermal units operate at the market price within its technological limits. For the ones, which generate their minimum (or maximum) power, they operate, respectively, at *i*-bus price:

$$\rho_i = f_{pi} \left[\frac{dC_i}{dP_{Gi}} + \sum_{k \in LIN} \sigma_k \frac{\partial P_k}{\partial P_{Gi}} \right] \qquad (7)$$

B. The competitive game

The ISO programs the operation with the objective of minimizing the costs of fuel. To do so, a unit commitment is run to determine the cheapest generating units. The decision of the generators to take part in the spot market to supply the load is reflected in the function of prices declared to the ISO. In this work, a non cooperative game is considered, where generating companies represent the players that try to maximize their profit using different pure strategies (the price that they declare for their production). Their benefits are given by:

$$B_k = \sum_{j \in \Omega_k} \left\{ \rho_j \times PG_i - \left[a_j + b_j PG_j + c_j PG_j^2 \right] \right.$$

Where Ω_i is the set of generating units belonging to the utility k and ρ_j is the market price at bus *j*. The generating costs are represented by:

$$C(PG) = a + bPG + cPG^2 (\$/h)$$

The payoff function of the player k is:

$$u_k : \prod_{i=1}^{\psi} S_i \rightarrow R$$

$$(S...............S_\psi) \rightarrow B_k$$

Where Ψ is the number of utilities, however, each generator knows only its own function of payment. The complete information is known by the ISO. Therefore, the economic behavior of the generators in the spot market can be analyzed by the ISO as a Static Game with Complete and Perfect Information.

IV. NUMERICAL RESULTS AND DISCUSSIONS

The power system used in this paper is a modified IEEE 30 bus system [17]. We assume that each player is a utility that can both supply its local and possibly sell power to the pool depending upon the market price.

Before transactions are defined, each utility supplies its local load by applying an economic dispatch. The characteristics of generators and generation levels without transactions are listed in Table 1. In Table 1, the price for no transactions λ is the marginal cost evaluated in P(i). Based on prices in Table 1 utilities A and C sell power in the grand coalition while utility B buys power.

Table 1: Generator Data

	Bus	Cost coefficients a(i)	b(i)	c(i)	Min [MW]	Max [MW]	P(i) [MW]	Λ [$/Mwh]
A	1	0	2	0.002	0	80	23.54	2.94
	2	0	1.75	0.0175	0	80	60.97	3.88
B	13	0	3	0.025	0	40	37.00	4.85
	23	0	3	0.025	0	30	19.20	3.96
C	22	0	1	0.0625	0	50	21.59	3.69
	27	0	3.25	0.00834	0	55	26.91	3.69

Participants are able to change their prices by adjusting the values of c_i. Using constrained economic dispatch, Pool benefits will be maximized when all participants trade power at marginal cost, m(i)=2c(i). As participants try to maximize their own benefits, they may either decrease their bids in order to sell more power or increase the price in order to earn more. Among the infinite set of feasible alternatives for each participant, we analyze the following three strategies:

H - Trade power at 1.15 times the marginal cost, m(i) = 2.3C(i). The participant's strategy is to *bid high.*

M - Trade power at marginal cost, m(i) = 2C(i).The participant's strategy is to *cooperate* with the Pool.

L - Trade power at 0.85 times the marginal cost, m(i) = 1.7C(i). The participant's strategy is to *bid low.*

In the example system, there are three possible coalitions among participants plus the grand coalition. Correspondingly, there are four possible non cooperative games between coalitions and counter-coalitions.

A. Transaction in a perfect competition

Let's assume first that no capacity limits are imposed on tie lines and participants are not facing any additional constraints (e.g., interruptible power from contracts, contingencies in local resources, etc.) in defining offered prices. We refer to this as a perfect competition, as participants cannot take advantage of external constraints. In the example, no participant has a biased control over the market price because of its size or location.

In Table 2, the payoff matrix of the non-cooperative game is shown for coalition ***S*** ={A,B} and its counter-coalition S^c={C}.In this game, utilities A and B agree to join forces in order to obtain higher benefits by combining strategies against utility C. The set of strategies available in each coalition is the product of the strategies for each participant in the coalition. Participants in the coalition have nine possible strategies: both bid at marginal costs (MM), both bid high (HH), both bid low (LL) in addition to (HM, HL, MH, ML, LH, and LM). An entry in the matrix is a pair of payoffs for coalition members and the counter-coalition, respectively. Each entry in the payoff matrix is computed using particular strategy (set of prices). The first value in the pair represents the sum of benefits in utilities A and B. The second value is the benefit obtained by utility C for the same combination of strategies.

Table 2: Payoff matrix [$/h]

{A-B} {C}	H	M	L
HH	20.30,0.27	20.34,0.24	20.38,0.20
HM	20.38,0.42	20.41,0.38	20.45,0.32
HL	19.97,0.65	19.98,0.59	19.99,0.56
MH	20.56,0.18	20.60,0.16	20.63,0.14
MM	20.62,0.30	20.65,0.28	20.69,0.23
ML	20.17,0.50	20.19,0.46	20.21,0.39
LH	20.46,0.10	20.49,0.09	20.53,0.07
LM	20.48,0.19	20.52,0.17	20.56,0.14
LL	19.95,0.34	19.98,0.32	20.01,0.27

While individual elements of the matrix are available to corresponding utilities, the entire matrix is accessible to the Pool coordinator. The characteristic function is an estimate of the best possible outcome in the worst situation for the coalition. In this example, the characteristic function of the coalition {A-B) is found by first locating the minimum benefit for coalitions in each row of Table 2(i.e.,$/h 20.3,20.38…19.97) maximum of selected minimums: v(A-B) = $/h 20.62. The chosen strategy is the max-min strategy. In this regard, utilities A and B bid at marginal costs because the bid offers the highest benefit when the other Pool participant (i.e. utility C) is minimizing the coalition's benefits. A similar analysis is made for utility C with v(C) = $/h 0.1; in this case, the max-min strategy for utility C is to bid high.

The characteristic function is a pessimistic estimate because it assumes that the counter coalition is playing to minimize the coalition's benefit, when in fact the counter coalition is trying to maximize its own benefits. For instance, in Table 2 if utility C decides to play M instead of its max-min strategy, the benefits of the coalition are higher than the characteristic function. Other criteria rather than the max-min criterion could be used to simulate the decision process in participants. For instance, both the pessimism-optimism index criterion and the criterion based on the principle of insufficient reason would concentrate the decision in a weighted combination of the best and the worst states for the participants. The choice of the adequate decision criteria will depend upon the characteristics of the actual participants playing the game in the Pool. The strategy MM for coalition {A-B} in Table 2 is said to be a dominant strategy because the coalition chooses the same strategy for different circumstances (strategies of the counter coalition) that the player faces. From Table 2 we learn that the row MM corresponds to the maximum benefit that coalition {AB} obtains for each column; hence, no matter which strategy utility C chooses, the best strategy for coalition {A-B} is MM. Accordingly, the strategy H is a dominant strategy in utility C.

If all bids are based on characteristic functions, the Pool will not reach the maximum system-wide benefit (i.e. $/h 20.93). For this analysis, utilities A and B will bid at marginal costs while utility C will bid high. The game is in equilibrium at this point, because the strategies are the best response to the opponent's strategy. The combination of dominant strategies (MM and H and the corresponding payoffs ($/h 20.62 and 0.30) are the dominant strategic equilibrium for the game. However, it is a rather non-stable equilibrium because there are other possible coalitions in which utilities may obtain higher benefits Table 3 gives the characteristic functions of coalitions in a perfect competition. The values for {A-B} and {C} are extracted from Table 2. All other entries are computed from the respective payoff matrices in the 4 possible games.

The grand coalition (fourth row in Table 3) is optimum from the Pool's perspective: load is supplied at minimum cost using available resources, and the maximum system-wide benefits are obtained. This solution is the same as that of a traditional centralized dispatch with minimizing cost functions.

Table 3: Payoff matrix [$/h]

Coalition	v(s)	V(sc)
{ A-C}	9.23	11.35
{A-B}	20.62	0.1
{B-C}	11.81	8.75
{A-B-C}	20.93	-

In this coalition, participants are obtaining $/h 9.11, 11.54 and 0.28, respectively. A coalition of utilities that is dominated through some other coalition would never become permanently established. There would be a tendency for the existing coalition to break up and be replaced by one that gives its members a larger share. Based on Table 3, we can infer which coalitions are likely to form. Clearly, none of the participants alone are doing as good as the grand coalition. For instance, utility B will obtain at least $/h 11.35 by playing against the possible coalition of utilities A and C (first row in Table 3), however, in the grand coalition utility B will obtain $/h 11.54. Therefore utility B, acting rationally, will probably decide to cooperate with the Pool. It is observed also that there are no economic reasons for any of the participants to desert the grand coalition and join other coalitions for higher benefits. This is also the case in Table 2 in which the game between {A-B} and {C} is in dominant strategy equilibrium; however, coalition {A-B} is a non stable coalition because its members are able to obtain higher benefits when joining the grand coalition. It is often claimed that the increasing pressure from competition in the energy market will help maximize the customers' benefits in a pool. This is supported by the results obtained from the game theory. In general, bigger the game, greater the variety of coalitions. The grand coalition of the game will be dominating as long as the market is not giving any participants or group of participants a relative advantage over the remaining participants. Hence, in the case of a game played by numerous participants there will be a narrower margin for possible coalitions against system-wide benefits.

B. Transactions in an imperfect competition

Several factors can alter the perfect competition described in the previous section, including the economic pre-eminence of some of the participants, the mix of generation resources available to each participant, the geographic situation of the participants, etc. When the market is in imperfect competition, some participants may find that it is possible to obtain higher benefits by colluding with a group of participants. In this section, we analyze the role of network constraints in a deregulated power pool. We suppose the line between buses 27 and 28 (linking utilities A and C) is on outage. In the previous case, utility C was exporting power using this line. It is cheaper now for utility C to import power than to supply the load locally. In this case, utility A is selling more power and utility B is buying less power in comparison with the case when the line was available. The characteristic functions for this case corresponding to all feasible coalitions are shown in Table 4.

Table 4: Payoff matrix [$/h]

Coalition	v(s)	v(sc)
{ A-C}	2.79	15.66
{A-B}	18.38	0.19
{B-C}	16.61	1.88
{A-B-C}	18.70	-

As line 27-28 is out, total generation costs are higher and the maximum system-wide benefit is $/h 18.70. In the grand coalition, participants obtain $/h 2.71, 15.83 and 0.16 respectively. In Table 4, utility C is doing better alone (bid high in this case) than joining the grand coalition.

From the second row in Table 4 utility C is guaranteed to obtain at least $/h 0.19 when playing against the possible coalition of utilities A and B. This amount is higher than the benefit obtained by utility C in the grand coalition (i.e. 0.16); hence, there is an incentive for utility C to defect the grand coalition. The coalition of utilities B and C is also likely to form. The strategy for coalition {B-C} in the third row of Table 4 is to allow B to bid below the marginal cost and C to bid above the marginal cost. Utilities B and C may find out that as they coordinate their bids, they obtain at least $/h 16.61 which is higher than that of the grand coalition. Hence, utilities B and C may decide to coordinate bids and share the extra benefits. In this case, utility A's benefit and that of the system-wide are lower than those obtained in the grand coalition. If we compare Tables 3 and 4, we learn that when line 27-28 is available, utilities B and C are obtaining the best result in the grand coalition (i.e. $/h 11.82). Hence, it will be advantageous for utilities B and C if line 27-28 is unavailable. In this case, even when system-wide benefits are lower, the individual benefits are higher. The Pool coordinator may use the procedures described in this section to identify unanticipated problems and take adequate corrective measures.

V. Conclusion

In this paper, a case was presented when conditions for perfect conditions are altered. The network imposes additional constraints on the bids, and participants increase their benefits by coordinating bid strategies and sharing benefits. Some participants may even prefer a more constrained network as they can take advantage of the situation and obtain higher benefits. The analysis may be used by Pool coordinators to identify noncompetitive situations and to encourage pricing policies that lead to maximum system-wide benefits. Participants in deregulated power pools can also use the specific aspects of the proposed analyses for price definition and decision make processes.

VI. References

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