Comparison between DFT, Adpative Window DFT and EDFT for Power Quality Frequency Spectrum Analysis

M. Caciotta, S. Giarnetti, F. Leccese
Faculty of Engineering
University Roma Tre
Rome, Italy
{caciotta,sgiarnetti,leccese}@uniroma3.it

Z.Leonowicz
Faculty of Electrical Engineering
Wroclaw University of Technology
Wroclaw, Poland
leonowicz@ieee.org

Abstract — **Spectral leakage and picket-fence effects associated with the system fundamental frequency variation and improperly selected sampling time window prevents a direct application of the DFT algorithm with a constant sampling rate. In particular it's very difficult to detect low level interharmonics and subharmonics. In this paper we compare two methods, proposed in literature, evaluating the detection capability and accuracy in frequency spectrum estimation. Several tests in different condition has been effected for the comparison.**

Keywords- Discrete Fourier Transform (DFT), Power Quality, spectral analysis, frequency estimation, spectral analysis.

I. INTRODUCTION

The constant growth of non linear systems connected to electric network is one of the principal causes of the distortion of the ideal electric sine wave, so producing a severe spectral content enrichment [1,2]. This last can be formed not only by components with frequencies integer multiple of the fundamental (harmonics), but also by components with frequencies not integer of the fundamental (interharmonics) or lower (subharmonics) [3]. The existence of these added spectral content deteriorates the quality of the electric power badly acting on the apparatus connected to the electric net. The effects can be multiple as: large load currents in the neutral wires of a 3 phase system causing overheating; overheating of standard electrical supply transformers shorten the life; additional losses in transmission lines, cables, generators, motors; in AC motors may arise undesired and difficulty manageable torques; poor power factor conditions that result in monthly utility penalty fees for major users with a power factor lee than 0.9; resonance that produces over-current surges that can destroyed capacitors and their fuses and damaged surge suppressors so causing an electrical system shutdown; false tripping of branch circuit breakers; skin effect that is normally ignored because it has a very little effect at power supply frequencies but, above the seventh harmonic (350 Hz), it will become significant causing additional loss and heating [4-7].

For these reasons an accurate identification of the signal spectral content is fundamental also to mitigate its effects.

The basic tool for these studies is still the Discrete Fourier Transform (DFT) that for harmonic analysis represents the most known and powerful algorithm - under the assumption of synchronous sampling [8]. DFT algorithm is able to accurately measure harmonics, but, if this condition is not satisfied its applicability is strongly reduced because its validity is strictly related to the analyzed waveform and the sampling window. In fact, if the width of DFT is not properly chosen or if a fixed window width is used uniformly for a spread of frequencies some negative aspects will arise. First of all the frequency resolution that is strictly dependent on the sampling frequency and the total number of samples, so to have a higher resolution it is necessary to apply a wider sampling window but this implies a strong stationarity of the signal. The second negative aspect is the spectral leakage that may reach an unacceptable level making sometimes blurs the true line spectrum [10,11].

Because the non ideal analyzed waveform, it is difficult to assure a synchronous sampling in power systems, so DFT algorithm will produce spectral leakage and picket fence effects. To face these problems in literature they are findable many algorithms and measuring techniques [12–15]; but, when the system fundamental frequency varies and harmonics/interharmonics are present in the measured waveforms, these approaches may suffer from low resolution accuracy and lower computational efficiency. For example, many studies are addressed to modify the FFT using windows and interpolations that can reduce the leakage and barrier effect, so improving the measuring accuracy of harmonics and interharmonics parameters [16], but the spectrum resolution is very low, therefore it cannot detect the interharmonics near the integer harmonics. For these purposes, the classical Hanning window in place of the rectangular window is often recommended [17].

Other authors think to face the problem using time-domain technique, e.g. in [18], an improved processing of harmonics and interharmonics by time-domain averaging is presented, but this method was developed on the assumptions that the fundamental frequency is known and that the synchronization of the sampling procedure is satisfied. In [19,20] the authors optimally summarizing the most common techniques, both in time and in frequency domain, underlying also their limits and so not suggesting a definitive methods.

Also Wavelet transform can be applied to detect interharmonics [21], but if the signals contain interharmonics close to harmonics, they cannot be separated accurately because of frequency band overlap of wavelets. Burg algorithm of Auto-Regressive model has higher resolution, but spectrum split may occur [22].

Other approaches use non-linear technique that use Phase-Locked Loop or Neural Networks. For example, in [23], a harmonic extraction and measurement method was presented based on a nonlinear, adaptive mechanism. The enhanced phase-locked loop extracts the amplitude and phase of the sinusoidal component of the input signal for which the internal operating point is preset while adaptively follows time variations in the characteristics of the signal. The proposed method requires several cycles (of 50/60 Hz) to settle to a steady-state and this limits its applications to cases for which the speed of measurement is not a critical factor. Moreover the frequency identification structure is strictly dependent to a priori knowledge of the input signal frequency. In [24,25,26] are shown some interesting applications obtained with Artificial Neural Network but the first is too specific, the second shows significant problems in the initial parameters setting that preventing easy algorithm application, the third is used only for subharmonics and still shows severe resolution limits.

In this paper we compare two DFT-based methods proposed in literature to find advantages and disadvantages with respect to the classical DFT in frequency spectrum estimation. The first one is based on the choice of the optimal number of signal points to perform a DFT with strongly reduced leakage. This method uses an adaptive window to generate two subsignals: the correlation between the two sequences is a period indicator. More detailed description of this algorithm can be found in chapter II.

The second analysed method use the Extended Discrete Fourier Transform (EDFT) to increase the spectrum frequency resolution. This algorithm is more complex than the first one but assures the best performances in the detection of low level harmonic and interharmonic components. More details on EDFT can be found in chapter III.

II. Easy Iterative Algorithm for Window Width Calculation

The proposed algorithm uses forecasting to precisely determine the most suitable window width for the DFT through an iterative procedure based on correlation calculation. The technique searches the most suitable window width in terms of local periodicity (or local similarity) of a signal sequence which complies with the periodicity requirement of DFT. The procedure is stopped when the correlation of two adjacent sections meets the defined termination criterion, before it the signal length of the first section is increased by one sample, and compared with the next adjacent section of the same length.

Consider a discrete-time signal S. The algorithm extracts two subsignal of length p:

$$X_p = \{S_1, S_2, S_3 \dots S_p\}$$

$$Y_p = \{S_{p+1}, S_{p+2}, S_{p+3} \dots S_{2p}\}$$

The correlation K_p between X_p and Y_p give an indication of signal periodicity with period p.

$$K_p = \frac{\langle X_p, Y_p \rangle}{\|X_p\| \|Y_p\|} \tag{1}$$

Where $\langle X_p, Y_p \rangle$ is the inner product between X_p and Y_p and $\|X_p\|$ and $\|Y_p\|$ are the norms of two signals.

If K_p value is 1 it is concluded that X_p and Y_p are identical, and p is the period of signal *S*. In general p varying iteratively and for each step the correlation coefficient is calculated. At the end of iteration we choose the p value that performs the higher K_p value.

To obtain the frequency spectrum it is performed a DFT on p samples of signal *S*. This method permits to limits the spectral leakage caused by a non-synchronous sampling of the signal. Moreover the presence of subharmonics and interharmonics change the period of signal that became different from the fundamental component period.

III. Interharmonics Detection Based On Iterative DFT

The frequency resolution of DFT is low when the sampling time t_c (it is also the width of rectangular window) is short because $f_r = 1/t_c = f_s/M$. The frequency resolution can be improved by increasing the number of frequency points.

The algorithm can be carried out by iterative DFT (IDFT) whose calculation procedure is as follows. At first, set the initial value of power spectrum vector W_0. Then, define the autocorrelation matrix R_1 as the Toeplitz matrix derived from the transform of W_0. Next, calculate the discrete spectrum S_1 and the power spectrum W_1 of the next iteration. The iteration will continue until the maximum iteration number is reached or the change of power spectrum (δ_p) is smaller than a given threshold.

Assume a signal has M points in time domain and it is necessary to obtain the desired frequency resolution to have N>M frequency points. W is the power spectrum vector whose initial value is $W_0 = [1,1 \dots 1]_{1\times N}$, R is the autocorrelation matrix, $E = e^{j2\pi T'F}$ is the kernel matrix of the transform. T and F are the times vector of the signal (M points) and the frequencies vector of the spectrum (N points). Then, the measuring process of interharmonics based on IDFT is expressed as follows:

$$R_n = E \cdot diag\left(\frac{W_{n-1}}{N}\right) \cdot E^H \tag{3}$$

$$S_n = \frac{XR_n^{-1}E.}{\left(I_{1\times M}\left(E^{*}.\times\left(R_n^{-1}E\right)\right)\right)} \tag{4}$$

$$W_n = S_n.\times S_n^{*} \tag{5}$$

$$\delta_p = \frac{sum(W_n) - sum(W_{n-1})}{sum(W_n)} \tag{6}$$

where *diag(X)* puts the elements of row vector *X* on the main diagonal to form a diagonal matrix, $\{\cdot\}^{-1}$denotes the inverse operation of matrix, $\{\cdot\}^*$ denotes the conjugate operation of matrix, $\{\cdot\}'$ represents the transpose operation of matrix, $\{\cdot\}^H$ represents the complex conjugate and transpose operation of matrix, $\{\cdot\}_n$ denotes the iteration number, $.\times$ and $./$ represents element-by-element multiplication and element-by-element division of two matrices with the same size respectively, $sum(W_n)$ is the addition of all elements of row vector , and $I_{1\times M}$ is a row vector with the size of $1 \times M$ and the elements of all one. Pre-multiplying a matrix by it, $I_{1\times M}$ can add the elements of every line of the matrix to form a row vector. Amplitude and phase spectrum can be calculated by:

$$A_m = 2\,X(f_m)$$

$$\theta_m = arg\big(X(f_m)\big) + \frac{\pi}{2}$$

IV. Simulations

The first simulation has been realized with a signal composed of six sinusoidal components. There are the fundamental with frequency 50.1 Hz, two even harmonics (third and fifth with amplitude equal to 3% and 2% respectively than the fundamental) a 20 Hz subharmonic component with amplitude 0.2% and two interharmonics equal to 82.3 Hz and 178.7 Hz with amplitude equal to 0.5% and 0.3%. The sampling frequency used is 10 kHz for a sampling time of 0.2 s.

Table 1 shows the amplitude and frequency estimations obtained by the two algorithms compared with DFT results for each component of the analyzed signal. The parameter N used for EDFT is equal to 100,000, able to assure a resolution of 0.1 Hz. Figure 1 shows a part of the spectrum for underlying the performance of the algorithms in the ability to detect the low amplitude components.

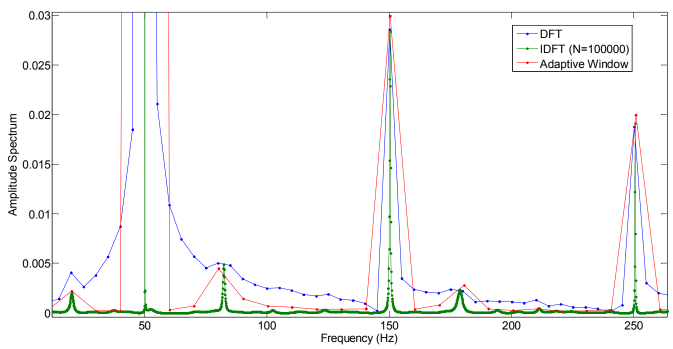

Figure 1. First simulation, part of the spectrum of the analyzed signal that shows the different accuracy in the recognition of low amplitude components provides by the three tested algorithms.

It is possible to see as, because of the leakage, the DFT is not able to clearly determine the interharmonics with frequencies 82 Hz and 178 Hz. Moreover, it is possible to note that the adaptive window algorithm, in spite of the fact that it has a resolution lower than DFT, is able to determine all the components also if the frequency and amplitude values are not very precise. The EDFT results show the effective resolution improvement and the accuracy in the determination of the spectral components.

TABLE I. First Test: Amplitude and Frequency Estimations

	Real Values	DFT	Adaptie Window	EDFT
Comp 1	20	20.01 (0.05%)	20.06 (0.3%)	20.2 (1%)
	0.002	0.004 (100%)	0.0022 (11.6%)	0.002 (-0.65%)
Comp 2	50.1	50.03 (-0.14%)	50.15 (0.01%)	50.1 (0%)
	1	0.998 (-0.15%)	1 (0%)	1 (0%)
Comp 3	82.3	n.d.	80.2 (-2.5%)	82.3 (0%)
	0.005	n.d.	0.00448 (-10.4%)	0.00489 (-2.12%)
Comp 4	150.3	150.1 (-0.13%)	150.5 (0.133%)	150.3 (0%)
	0.03	0.02859 (-4.7%)	0.03 (-0.1%)	0.02839 (-5.37%)
Comp 5	178.7	n.d.	180.5 (1.01%)	178.8 (0.06%)
	0.003	n.d.	0.0028 (-7.3%)	0.0023 (-21.8%)
Comp 6	250.5	250.1 (-0.16%)	250.8 (0.12%)	250.5 (0%)
	0.02	0.019 (0.019%)	0.02 (-0.25%)	0.019 (-4.35%)

For the second test has been used the same signal of the previous one but it has been halved the observation window reducing the time to 0.1 s. Table 2 show the results obtained applying the three methods.

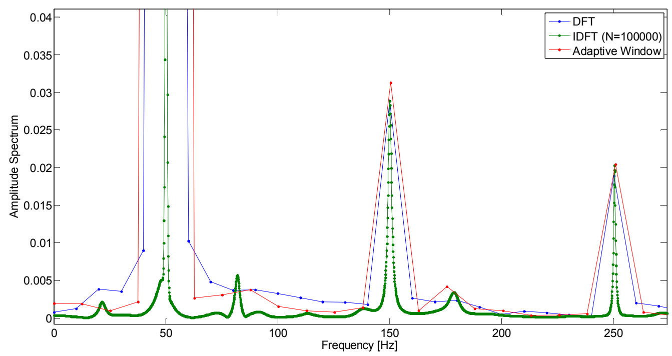

Figure 2.Second simulation, part of the spectrum of the analyzed signal that shows the different accuracy in the recognition of low amplitude components provides by the three tested algorithms.

In this case the adaptive window is not able to determine the 20 Hz subharmonic while the DFT shows the same problems as in the previous test. The EDTF algorithm keeps the same resolution and it is still able to determine all the components. The accuracy of the frequency and amplitude estimation is lower than in previous case. Moreover, in the EDTF it is possible to note spurious oscillations close to spectral components. A further resolution reduction of the observation window could result in mixing of oscillations with the frequency components with lower amplitude.

TABLE II. SECOND TEST: AMPLITUDE AND FREQUENCY ESTIMATIONS

	Real Values	DFT	Adaptive Window	EDFT
Comp 1	20 Hz	20.02 (0.1%)	n.d.	21.5 (7.5%)
	0.002	0.003829 (91.4%)	n.d	0.0021 (5%)
Comp 2	50.1Hz	50.05 (-0.1%)	50.19 (0.18%)	50.1 (0%)
	1	0.9993 (-0.07%)	1.001 (0.1%)	0.9998 (-0.02%)
Comp 3	82.3Hz	n.d.	87.83 (6.72%)	82 (-0.36%)
	0.005	n.d.	0.0037 (-26%)	0.0057 (14%)
Comp 4	150.3Hz	150.2 (0.066%)	150.6 (0.2%)	150.2 (-0.06%)
	0.03	0.02884 (3.87%)	0.03128 (4.27%)	0.02888 (-3.73%)
Comp 5	178.7Hz	n.d.	175.7 (-1.68%)	178.7 (0%)
	0.003	n.d.	0.00413 (37.7%)	0.0034 (13.3%)
Comp 6	250.5Hz	250.3 (0.08%)	250.9 (0.16%)	250.6 (0.04%)
	0.02	0.0189 (-5.5%)	0.0204 (2%)	0.0202 (1%)

Figure 3 shows a detail of the spectrum given by the EDFT with different values of N. As it is possible to see the spectrums do not show great difference for *N* bigger than 5,000.

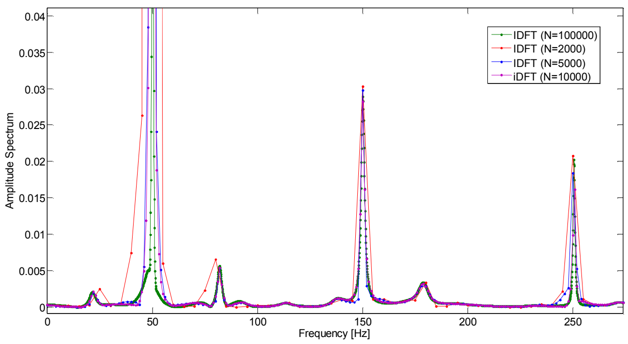

Figure 3.Third simulation, part of the spectrum of the analyzed signal that shows the different accuracy in the recognition of low amplitude components provides by the different value of N for the EDFT algorithm.

V. CONCLUSIONS

Two methods for frequency estimation in power quality signal distortions were analyzed. Several test has been done to compare the accuracy of the detection of all spectral components. The levels of subharmonics and interharmonics components inserted in the test signals are very low (between 0.2% and 0.5%). In this way we want to simulate realistic situations in voltage supply waveforms. The test was also repeated using a shorter signal.

The first method is based on an adaptive window algorithm to search the samples number of the signal to perform best using standard DFT in terms of leakage. Moreover the algorithm limits the effects of an asynchronous sampling of the signal.

The second method is based on iterative DFT able to increase the frequency resolution in the spectrum.

The first method assures good capability in component detection but requires long observation time of the signal. In shorter observation only one component was not detected. The second method shows high performances in all cases. In this test high accuracy is guaranteed with 5,000 points of frequency spectrum. In terms of computational complexity the adaptive window method is very lightweight. The computational complexity of IDFT depends on the desired frequency resolution but in general is higher than in the case of the adaptive window algorithm.

REFERENCES

[1] Caciotta M., Leccese F., Trifirò T.: “From Power Quality To Perceived Power Quality” presented to The IASTED International Conference on Energy and Power Systems EPS 2006, 29-31 March 2006, Chiang Mai, Thailand, Proceedings 526-119, pp. 94-102, ISBN CD: 0-88986-586-8.

[2] J. Arrillaga, N.R. Watson, *Power System Harmonics, Second Edition,* John Wiley & Sons Ltd, 2003, ISBN: 0-470-85129-5.

[3] CEI EN 50160: Voltage characteristics of electricity supplied by public distribution systems.

[4] J. Arrillaga, N.R. Watson, S. Chen, *Power System Quality Assesment*, John Wiley & Sons, 2000, ISBN: 0-471-98865-0.

[5] J.P.G. de Abreu, A. E. Emanuel, "The need to limit subharmonic injection,” Proceedings of the 9th International Conference on Harmonics and Quality of Power, Vol. I, October 1-4, 2000, pp. 251-253.

[6] E.F. Fuchs, “Power quality in power systems and electric machines,” ECEN 5787 Course Notes, Department of Electrical and Computer Engineering. University of Colorado at Boulder, Fall 2005.

[7] J.F. Fuller, E.F. Fuchs, D.J. Roesler, "Influence of harmonics on power system distribution protection," *IEEE Transactions on Power Delivery,* April 1988, vol. TPWRD-3, no. 2, pp. 546-554.

[8] Zhu T. X.: ‘Exact Harmonics/Interharmonics Calculation Using Adaptive Window Width,’ IEEE Transactions on Power Delivery, vol. 22, No. 4, october 2007.

[9] R. W. Ramirez, The FFT: Fundamentals and Concepts. Englewood Cliffs, NJ: Prentice-Hall, 1985.

[10] E. O. Brigham, The Fast Fourier Transform and its Applications. Englewood Cliffs, NJ: Prentice-Hall, 1988.

[11] Akke, M.: ‘Frequency estimation by demodulation of two complex signals’, IEEE Trans. Power Deliv., 1997, 12, (1), pp. 157–163.

[12] Lobos, T., Kozina, T., and Koglin, H.-J.: ‘Power system harmonics estimation using linear least squares method and SVD’, IEE Proc., Gener. Transm. Distrib., 2001, 148, (6), pp. 567–572.

[13] Dash, P.K., Panda, S.K., Mishra, B., and Swain, D.P.: ‘Fast estimation of voltage and current phasors in power networks using an adaptive neural network’, IEEE Trans. Power Syst., 1997, 12, (4), pp. 1494–1499.

[14] Saiz, V.M.M., and Guadalupe, J.B.: ‘Application of Kalman filtering for continuous real-time tracking of power system harmonics’, IEE Proc., Gener. Transm. Distrib., 1998, 144, (1), pp. 13–20.

[15] Aghazadeh, R., Lesani, H., Sanaye-Pasand, M., and Ganji, B.: ‘New technique for frequency and amplitude estimation of power system signals’, IEE Proc., Gener. Transm. Distrib., 2005, 152, (3), pp.435–440.

[16] Qi Caijun, Wang Xiaohai, "Interharmonics Estimation Based on Interpolation FFT Algorithm", Transactions of China Electrotechnical Society, Vol.I8 , No. I, pp. 92-95, Feb 2003.

[17] Testa A., Gallo D., Langella R., "On the Processing of Harmonics and Interharmonics: Using Hanning Window in Standard Framework", IEEE Transactions on Power Delivery, Vol. 19, No.1, pp. 28-34, Jan 2004.

[18] Z. Liu, J. Himmel, and K. W. Bonfig, "Improved processing of harmonics and interharmonics by time-domain averaging," IEEE Trans.Power Del., vol. 20, no. 4, pp. 2370–2380, Oct. 2005.

[19] Thomas, D.W.P., and Woolfson, M.S.: 'Evaluation of frequency tracking methods', IEEE Trans. Power Deliv., 2001, 16, (3), pp. 367–371.

[20] Begovic, M.M., Djuric, P.M., Dunlap, S., and Phadke, A.G.: 'Frequency tracking in power networks in the presence of harmonics', IEEE Trans. Power Deliv., 1993, 8, (2), pp. 480–486.

[21] Xue Hui, Yang Ren-gang, " A novel method for non-integer harmonics measurement using continuous wavelet transform", Automation of Electric Power Systems, VoI.27, No.5, pp.49-53, May 2003.

[22] Chen Han, Liu Huijin, Li Dalu: 'Interharmonics Spectral Estimation Based on Improved Burg Agorithm,'

[23] M. Karimi-Ghartemani and M. R. Iravani, "Measurement of harmonics/ interharmonics of time-varying frequencies," IEEE Trans. Power Del., vol. 20, no. 1, pp. 23–31, Jan. 2005.

[24] Lin, H.C.: 'Intelligent neural-network-based adaptive power-line conditioner for real-time harmonics filtering', IEE Proc., Gener. Transm. Distrib., 2004, 151, (5), pp. 561–567.

[25] Dash, P.K., Swain, D.P., Routray, A., and Liew, A.C.: 'Harmonic estimation in a power system using adaptive perceptrons', IEE Proc., Gener. Transm. Distrib., 1996, 143, (6), pp. 565–574.

[26] Leccese F.: "Subharmonics Determination Method based on Binary Successive Approximation Feed Forward Artificial Neural Network: a preliminary study", 9th International Conference on Environment and Electrical Engineering- EEEIC 2010, 16-19 March 2010, Prague, Czech Republik, ISBN:978-1-4244-5371-9.