WLS State Estimation in Polar and Rectangular Coordinate Systems for Power System with UPFC: Significance of Types of Measurements

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Abstract—**The paper presents a comparative investigations of the state estimation of a power system with UPFC in the polar and rectangular coordinate systems, paying special attention to types of utilized measurements. The estimation with the use of the weighted-least-squares method is considered. During investigations, impact of a coordinate system, in which the state vector is considered, stress of a power system, a data redundancy, and types of utilized measurements on such parameters of a state estimation as: the number of iterations, the condition number of the gain matrix, parameters characterizing accuracy of calculation results is analyzed.**

Keywords-power system, UPFC, state estimation, polar coordinate system, rectangular coordinate system

I. INTRODUCTION

Monitoring and control of a power system require possessing of the credible assessment of states of this system. In modern control centers, such assessment is produced with the use of state estimation. The state estimation enables obtaining the best estimate of the power system state vector (magnitudes and phase angles of bus voltages) utilizing a set of measurement data, which usually comprises of (i) active and reactive power injections, (ii) active and reactive power flows, (iii) bus-voltage magnitudes. Short time of calculations and high credibility of the estimation results belong to the most important features of the power system state estimator.

There are many papers dealing with power system state estimation. Some of these papers present comparative analyses of the different estimation methods, e.g. [1], [2]. Also, it is possible to find some papers, which present different approaches to state estimation of power system with embedded FACTS [8]-[13] However, there is no paper in which quantitative dependence of features of the power system state estimation from such factors as the power system load and also a level of data redundancy, especially when in a power system there is UPFC, is described. The purpose of the paper is to present results of investigations of the mentioned dependence for the polar and rectangular coordinate systems when attention is paid also to types the utilized measurement.

In the paper, the following assumptions are taken into account: (i) the power system state is calculated using Weighted Least Squares (WLS) estimator, (ii) the state vector is considered in the polar and rectangular coordinate systems, (iii) analyzed features of the estimator are number of iterations, condition number of the gain matrix, and accuracy of the estimation results.

II. THE WEIGHTED LEAST SQUARES POWER SYSTEM STATE ESTIMATOR

A power system state estimator based on the WLS method [3] - [5] is the most known. The estimator assumes minimizing the following objective function:

$$J(\mathbf{x}) = \frac{1}{2}[\mathbf{z} - \mathbf{h}(\mathbf{x})]^{\mathrm{T}} \mathbf{R}^{-1} [\mathbf{z} - \mathbf{h}(\mathbf{x})], \tag{1}$$

where: $\mathbf{x}$ – a power system state vector, $\mathbf{z}$ – a vector of measurements, $\mathbf{h}(\mathbf{x})$ – a vector of nonlinear functions, representing dependence of measured quantities from the state vector, $\mathbf{R}$ – a diagonal matrix of measurement covariances.

Solving the following normal-equation set we find out a solution of the estimation problem:

$$\mathbf{G}(\mathbf{x}^k) \cdot (\mathbf{x}^{k+1} - \mathbf{x}^k) = -\mathbf{g}(\mathbf{x}^k), \tag{2}$$

where: k – a number of iteration, $\mathbf{x}^k$ – a solution vector at the *k-th* iteration

$$\mathbf{G}(\mathbf{x}^k) = \mathbf{H}^T(\mathbf{x}^k) \cdot \mathbf{R}^{-1} \cdot \mathbf{H}(\mathbf{x}^k), \tag{3}$$

$$\mathbf{H}(\mathbf{x}) = \frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}, \tag{4}$$

$$\mathbf{g}(\mathbf{x}) = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} = -\mathbf{H}^T(\mathbf{x})\mathbf{R}^{-1}[\mathbf{z} - \mathbf{h}(\mathbf{x})]. \quad (5)$$

G(x) is called a gain matrix. It is a sparse, positive determined and symmetric matrix for a fully observable power system.

III. The Power System State Estimator in the Polar and Rectangular Coordinate Systems

A power system state vector can be considered in the polar and rectangular coordinate system. In the polar coordinate system, the voltage $\overline{V}_i$, i.e. the bus voltage at the *i*-th bus is considered in the form $\overline{V}_i = V_i e^{j\delta_i}$, where V_i, δ_i – a magnitude and phase angle of the voltage respectively, and a state vector is presented as follows:
$\mathbf{x} = [\delta_2, \delta_3, \ldots \delta_n, V_1, V_2, \ldots V_n]^T$, where: δ_i $i = 2, 3, \ldots, n$ - phase angles of voltages at the buses 2, 3, …, *n*, V_i $i = 1, 2, \ldots, n$ – magnitudes of voltages at the buses 1, 2, …, *n*.

The bus 1 is considered as a reference bus. The phase angle for the reference bus is equal to zero.

In the rectangular coordinate system $\overline{V}_i = e_i + \mathrm{j} f_i$, where e_i, f_I –- a real and imaginary part of the voltage respectively, and a state vector is presented as
$\mathbf{x} = [e_1, e_2, \ldots e_n, f_2, f_3 \ldots f_n]^T$, where: e_i $i = 1, 2, \ldots, n$ – real parts of voltages at the buses 1, 2, …, *n*, f_I – imaginary parts of voltages at the buses 2, 3, …, *n*.

The imaginary part of the voltage for the reference bus is equal to zero.

For both the coordinate systems we can write the following relationships among measured quantities and elements of state vector

$$P_i - jQ_i = V_i^* \mathbf{Y}_{row\,i} \mathbf{V} \quad , \quad (6)$$

$$P_{ij} - jQ_{ij} = \left[-(\overline{y}_{si} + \overline{y}_{ij}) \quad \overline{y}_{ij}\right] \cdot \left[V_i^2 \quad \overline{V}_j \cdot \overline{V}_i^*\right]^T, \quad (7)$$

where: P_i, Q_i – an active and reactive injection at the *i*-th bus respectively, P_{ij} Q_{ij} – an active and reactive power flow between the *i*-th and the *j*-th bus respectively, y_{ij} – an admittance of the series branch connecting the *i*-th and the *j*-th bus, y_{si} – an admittance of the shunt branch at the *i*-th bus, $\mathbf{Y}_{row\,i}$ – the *i*-th row of an admittance matrix,
$\mathbf{Y}_{row\,i} = [\overline{y}_{i1}, \overline{y}_{i2}, \ldots, \overline{y}_{in}]$, $\mathbf{V} = [\overline{V}_1, \overline{V}_2, \ldots, \overline{V}_n]^T$.

The relationships (6) and (7) are base for determination of elements of the function vector **h**(**x**).

The most important features of the estimator in the polar coordinate system is existence transcendental functions in the objective function (1). For these functions, the Taylor series is an infinite one.

In the rectangular coordinate system, we take into account the formulas, in which there are only quadratic terms. This fact leads to significant simplification of an expansion in Taylor series for *J*(**x**) in the rectangular coordinates.

IV. A Model of UPFC in the Polar and Rectangular Coordinate Systems

The UPFC device in its general form can provide simultaneous real-time control of all basic power system parameters (voltage, voltage phase-angle, and impedance), or any their combinations and in consequence, determining the transmitted power.

UPFC consists of two switching converters. These converters are voltage sourced inverters using gate turn-off thyristor valves. They are coupled with common DC-link (which is a DC storage capacitor) which allows free exchanging real power between the inverters *Inverter 1* and *Inverter 2*. *Inverter 1* is connected with a transmission line through shunt transformer. It supplies or absorbs a real power to/from *Inverter 2* by the common DC link. *Inverter 2* is coupled with the transmission line through a series transformer. It provides its principle function by injecting an AC voltage with a controllable magnitude and a phase angle. Each inverter can independently generate or absorb reactive power at its own AC output terminal.

The model of UPFC shown in Fig. 1 according to [7] is composed of two controllable ideal voltage-sources. These two coordinated synchronous voltage sources represent the UPFC adequately for the purpose of fundamental-frequency steady-state analysis.

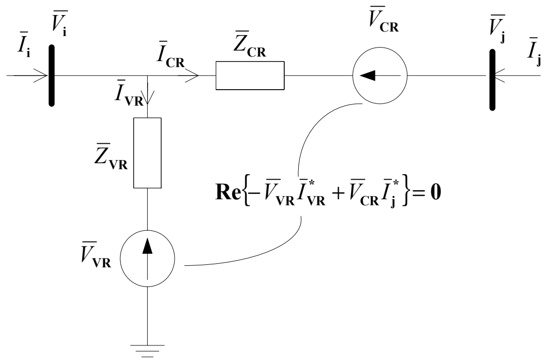

Figure 1. Equivalent model of UPFC.

Source voltages of the sources in the model of UPFC can be expressed as follows: $\overline{V}_{VR} = V_{VR} e^{j\delta_{VR}}$, $\overline{V}_{CR} = V_{CR} e^{j\delta_{CR}}$, for the polar coordinate system, and $\overline{V}_{VR} = e_{VR} + \mathrm{j} f_{VR}$, $\overline{V}_{CR} = e_{CR} + \mathrm{j} f_{CR}$ for the rectangular coordinate system.

Taking into account the equivalent circuit shown in Fig. 1 we can derive the matrix equations:

for bus *i*

$$P_i - jQ_i = [(\overline{y}_{VR} + \overline{y}_{CR}) \quad -\overline{y}_{CR} \quad -\overline{y}_{CR} \quad -\overline{y}_{VR}] \cdot \left[V_i^2 \quad \overline{V}_j \cdot \overline{V}_i^* \quad \overline{V}_{CR} \cdot \overline{V}_i^* \quad \overline{V}_{VR} \cdot \overline{V}_i^*\right]^T, \quad (8)$$

for bus j

$$P_j - jQ_j = \left[-\bar{y}_{CR} \quad \bar{y}_{CR} \quad \bar{y}_{CR}\right]\cdot\left[\bar{V}_i\cdot\bar{V}_j \quad V_j^2 \quad \bar{V}_{CR}\cdot\bar{V}_j^*\right]^T, \qquad (9)$$

for series inverter

$$P_{CR} - jQ_{CR} = \left[-\bar{y}_{CR} \quad \bar{y}_{CR} \quad \bar{y}_{VR}\right]\cdot\left[\bar{V}_i\cdot\bar{V}_{CR}^* \quad \bar{V}_j\cdot\bar{V}_{CR}^* \quad V_{CR}^2\right]^T, \qquad (10)$$

for shunt inverter

$$P_{VR} - jQ_{VR} = \left[-\bar{y}_{VR} \quad \bar{y}_{VR}\right]\cdot\left[\bar{V}_i\cdot\bar{V}_{VR}^* \quad V_{VR}^2\right]^T, \qquad (11)$$

where:
$\bar{y}_{CR} = \bar{Z}_{CR}^{-1}$, $\bar{y}_{VR} = \bar{Z}_{VR}^{-1}$.

Neglecting UPFC losses, we can state that UPFC cannot absorb and injects real power, i.e. the active power supplied to the shunt converter P_{VR} equals the active power demanded by the series converter, P_{CR}:

$$P_{bb} = P_{VR} + P_{CR} = 0, \qquad (12)$$

V. The Considered Parameters of Power System State Estimation

A. Number of Iterations in a State Estimation Process

Number of iterations in a state estimation process is essential factor which influences on time duration of an estimation process. It is related to the convergence of estimation calculations.

B. Accuracy of the State Estimation

In [6], accuracy of state estimation is evaluated with the use of the ratio J_e/J_M, where:

$$J_M = \frac{1}{m}\sum_{i=1}^{m}\left[\left(z_i - z_i^r\right)/\sigma_i\right]^2, \quad J_e = \frac{1}{m}\sum_{i=1}^{m}\left[\left(\hat{z}_i - z_i^r\right)/\sigma_i\right]^2, \qquad (13)$$

z_i, $\hat{z}_i$, z_i^r - the measured, estimated and real value of the i-th measured quantity, respectively.

J_M is a global measure of discrepancy among measured and real values of the measured quantities and J_e is a global measure of discrepancy among estimated and real values of the mentioned quantities. If $J_e/J_M < 1$, then results of estimation are more precise than measurement data.

To determine the ratio J_e/J_M only measured quantities are taken into account, other quantities are neglected. This fact is a drawback of the presented measure of the accuracy of state estimation.

Sometimes there is $J_e/J_M < 1$, but estimated values of the non-measured quantities may be burden with a non-acceptable errors, especially when redundancy is low.

The following measure of accuracy of state estimation has no the earlier described drawback:

$$N_{3\sigma} = M_{p\_q,3\sigma}/M_{p\_q}, \qquad (14)$$

where M_{p_q} – a number of all power flows and power injections; $M_{p\_q,3\sigma}$ – a number of the power flows and power injections, for which estimated values differ from real values less than 3 standard deviations.

When the ratio $N_{3\sigma}$ is used, the following assumptions are adopted: (i) all power flows and power injections are estimated on the base of the earlier estimated bus voltages, (ii) for each power flow and power injection, we have knowledge about the standard deviation of the normal distribution describing small errors, which would have burdened measurement data of this quantity if it has measured.

Both the presented measures of accuracy of state estimation can be used only when a state estimation method is tested and real values of distinguished quantities are known. When states of an actual power system are estimated the mentioned measures cannot be considered.

C. Conditioning of a Power System State Estimation Process

A measure of conditioning of a power system state estimation process is the condition number of the gain matrix. In the paper, the mentioned Condition Number is defined as:. $\text{cond}(\mathbf{G}) = |\lambda_M|/|\lambda_m|$, where: λ_m, λ_M – the minimal and maximal (by moduli) eigenvalue of **G**, respectively.

The greater the condition number is the worse conditioned a power system state estimation process is.

If the estimation process is ill-conditioned then measurement errors have significant influence on computational process. If the condition number is large, even a small errors in measurement data may cause large errors in a state vector. The ill-conditioning of the estimation process often leads to a worse convergence of the process or even to lack of the convergence of this process. The reasons of ill-conditioning can be large differences in values of the elements of the matrix **R**, existence long and short lines connected with the same bus or existence virtual measurements.

VI. Description of a Realization of Investigations

For each considered coordinate system, the investigations have been performed under the following assumptions:

1. The IEEE-14-bus test system is used.
2. UPFC devices is installed on the line between the bus 6 and the bus 12, and this device compensates line reactance.
3. 100 load patterns are taken into account. Each load pattern is obtained from the base load flow by appropriate scaling power injections. The factor used for

the scaling ranges from 0.3182 to 2.1182 with the step equal to 0.0182. Such the scaling ensures monotonic changes of power losses in the test system.

4. Each power flow and each power injection is modified by adding to it the different pseudorandom number representing the small error characterized by the Gaussian distribution with the mean equal to 0 and a standard deviation σ, where:
$\sigma = 1/3\left[(0.001+0.0025)FS + 0.02M\right]$ for active power;
$\sigma = 1/3\left[(0.001+0.005)FS + 0.02M\right]$ for reactive power and
$\sigma = 1/3\left[(0.0005+0.0025)FS + 0.003M\right]$ for absolute value of voltage, *FS* - a measurement scope, *M* – a measured value. For each load pattern, 600 cases of generation of the mentioned pseudorandom numbers is considered.
5. For each load pattern, 60 cases of arrangement of measuring systems in the test system is tested. The specific arrangement of measuring systems is selected randomly.
6. In order to investigate influence of types of measurements on estimation results, all cases of arrangement of measuring systems were divided into 6 groups depending on number of measurements of each type. Tab. I and Tab. II, presents numbers of measurements of the selected types in the particular groups.
7. The data redundancy *r* is equal to 1.33 (43 measurements) and 3.03 (100 measurements).
8. For evaluation of the stress of a test system the following *Load Factor* is introduced:

$$Load\ Factor = \sum_{i=1}^{n} P_i \Bigg/ \sum_{i=1}^{m} P_i^+ , \qquad (15)$$

where: P_i^+ - a positive power injection.

Load Factor represents active power losses in the considered power system. During the investigations, *Load Factor* has changed from 0.0177 to 0.1283. The larger that factor is the more loaded power system is.

VII. The Results of the Investigations

In this Section, the considered parameters versus the parameter *Load Factor* are analyzed. In one figure, the results of the investigations for the polar and rectangular coordinate systems are plotted. In Fig. 2 – Fig. 9, results of investigations, in which types of measurements are not distinguished, are presented. Types of measurements are taken into account in investigations, of which results are in Tab. III – Tab. VIII.

In Fig. 2 and Fig. 3, an average number of iterations in the estimation process is shown. The result of investigations is that the mentioned number of iterations depends on a stress of the power system. More iterations are required for a larger stress. It can be noticed, that the estimation process in the polar coordinate system usually requires more iterations than in the rectangle coordinate system. The data redundancy has an insignificant impact on the considered number of iterations. Also, types of utilized measurements have an insignificant impact on a number of iterations.

TABLE I. Numbers of measurements for r = 1.33

Types of measurements	Measurement case					
	1	2	3	4	5	6
V	2	2	2	2	6	10
P_{bus}/Q_{bus}	1/1	4/4	9/9	13/13	1/1	1/1
P_{flow}/Q_{flow}	17/17	14/14	9/9	5/5	15/15	13/13
$P_{zeroinj}/P_{zeroinj}$	2/2	2/2	2/2	2/2	2/2	2/2
P_{UPFC}	1	1	1	1	1	1

TABLE II. Number of measuremnts for r = 3.03

Types of measurements	Numbers of cases					
	1	2	3	4	5	6
V	5	5	5	5	7	15
P_{bus}/Q_{bus}	6/6	7/7	9/9	13/13	6/6	6/6
P_{flow}/Q_{flow}	39/39	38/38	36/36	32/32	38/38	34/34
$P_{zeroinj}/P_{zeroinj}$	2/2	2/2	2/2	2/2	2/2	2/2
P_{UPFC}	1	1	1	1	1	1

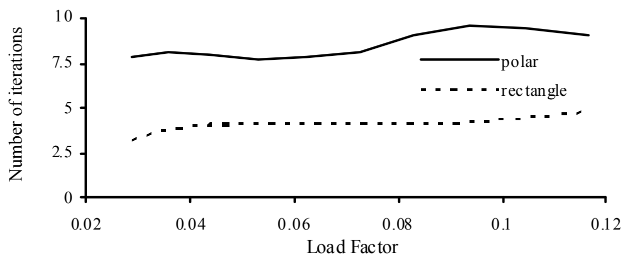

Figure 2. Number of iterations for r = 1.33.

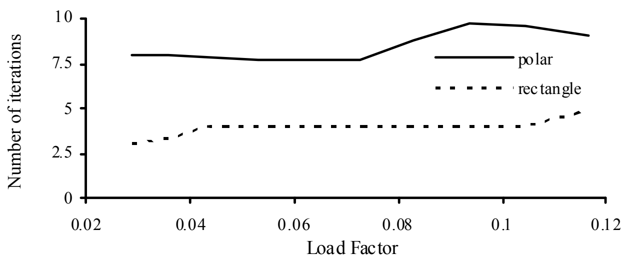

Figure 3. Number of iterations for r = 3.03

The ratio $N_{3\sigma}$ versus *Load Factor* is shown in Fig. 4 and Fig. 5. Analyses of the ratio $N_{3\sigma}$ (accuracy of a state estimation) point out, that the data redundancy and *Load Factor* have a significant influence on accuracy of results of the state estimation calculations. When the data redundancy is low and stress of the power system is larger then the state estimation can give not enough accurate results. For both the considered coordinate systems accuracy of estimation results is almost the same. Tab. III and Tab. IV present influence of types of measurements on accuracy of the estimation process. In these tables, for each value of *Load Factor*, a value of the coefficient of variation (C. of Var.) is presented. C. of Var. is calculated dividing mean deviation of the considered quantity by the mean value of the mentioned quantity. It can be seen, that power measurements especially power-injection measurements have a favorable influence on accuracy of the power system state estimation. The mentioned influence is larger (C. of Var. is larger) for lower data redundancy.

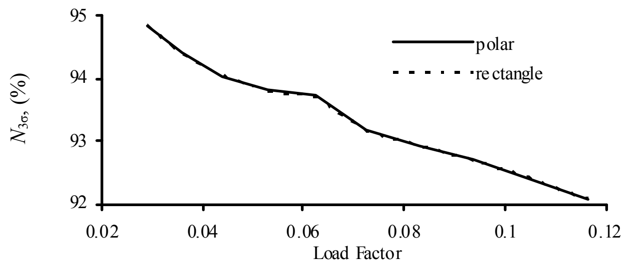

Figure 4. The ratio $N_{3\sigma}$ for r = 1.33.

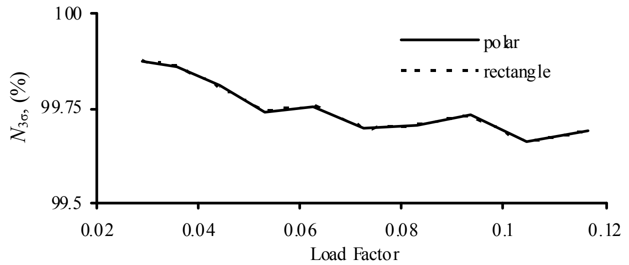

Figure 5. The ratio $N_{3\sigma}$ for the r = 3.03.

TABLE III. THE RATIO $N_{3\sigma}$ FOR R = 1.33

Load Factor	Measurement case						C.of Var. %
	1	2	3	4	5	6	
0.03	95.93	96.53	96.14	98.23	92.50	89.75	2.62
0.12	93.42	93.51	93.26	96.39	89.49	86.44	2.98

TABLE IV. THE RATIO $N_{3\sigma}$ FOR R = 3.03

Load Factor	Measurement case						C.of Var. %
	1	2	3	4	5	6	
0.03	99.83	99.89	99.91	100.0	99.86	99.76	0.06
0.12	99.56	99.69	99.77	100.0	99.65	99.47	0.13

Analyzing values of the condition number, we can ascertain that this parameter is much lower for the rectangular coordinate system than for the polar coordinate system (Fig. 6 – Fig. 9, Tab. V – Tab. VIII). The condition number as a function of *Load Factor* is plotted in Fig. 6 – Fig. 9. Also in the case of that parameter there is significant dependence from the data redundancy and *Load Factor*. It can be observed that in the first iteration, the condition number increases relatively slowly when *Load Factor* increases. It should be noted, that in the estimation calculations the flat start is used and in successive iterations, the matrix **G**(**x**) is modified. For the polar coordinate system, the condition number in the last iteration is higher for lower *Load Factor*. For the rectangular coordinate system there is no significant difference between the first and the last iteration.

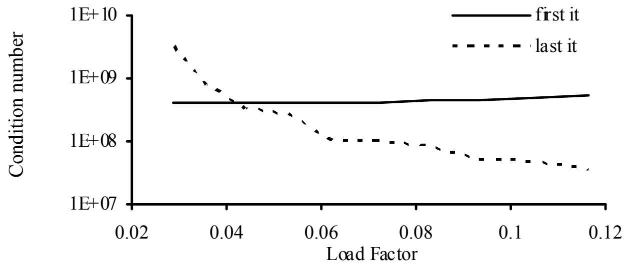

Figure 6. The condition number for the polar coordinate system for r = 1.33.

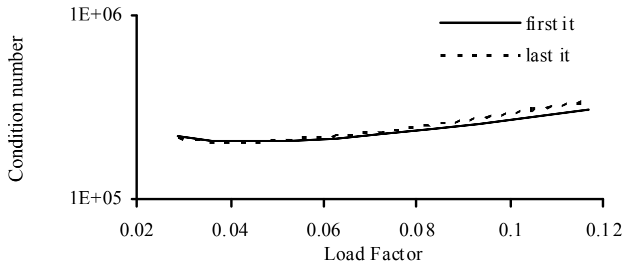

Figure 7. The condition number for the rectangular coordinate system for r = 1.33.

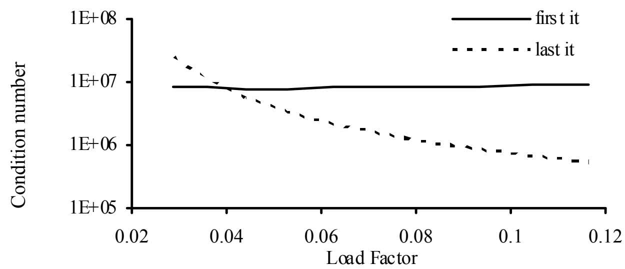

Figure 8. The condition number for the polar coordinate system for r = 3.03.

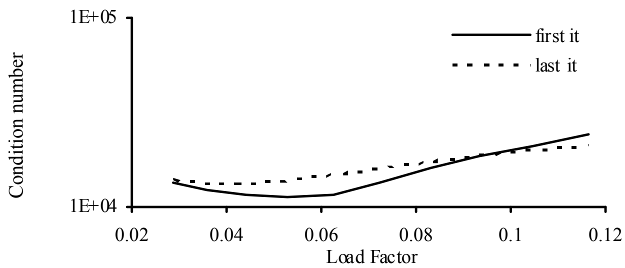

Figure 9. The condition number for the rectangular coordinate system for r = 3.03.

In many cases, independently of a coordinate system, when there are more power-injection measurements the condition number is larger (Tab. V – Tab. VIII). However, one can point out such cases, when this statement is not valid.

Considering C. of Var. of the condition number, one can note that this coefficient is very large for lower data redundancy. This fact means that impact of types of utilized measurements on results of state estimation is very essential for the pointed out data redundancy. The discussed impact is larger for the polar coordinate system than for the rectangular coordinate system. C. of Var. of the condition number is relatively low for larger data redundancy. In this case for the polar coordinate system C. of Var. of the condition number is smaller than for the rectangular coordinate system if values of *Load Factor* are small. For large values of *Load Factor*, the situation changes. For the rectangular coordinate system C. of Var. of the condition number is smaller than for polar coordinate system.

TABLE V. THE CONDITION NUMBER FOR THE POLAR COORDINATE SYSTEM FOR R = 1.33

It.	Load Factor	Measurement case						C.of Var. %
		1	2	3	4	5	6	
F. It.	0.03	2.2E07	1.1E08	3.3E07	2.1E09	1.6E08	5.5E07	136.02
L. It.	0.03	9.3E07	4.8E08	1.1E08	1.8E10	1.1E09	1.0E08	147.73
F. It.	0.12	2.3E07	1.5E08	3.5E07	2.5E09	3.8E08	9.1E07	123.95
L. It.	0.12	1.8E06	6.8E06	2.5E06	2.0E08	7.1E06	1.9E06	148.40

TABLE VI. THE CONDITION NUMBER FOR THE RECTANGULAR COORDINATE SYSTEM FOR R = 1.33

It.	Load Factor	Measurement case						C.of Var. %
		1	2	3	4	5	6	
F. It.	0.03	5.9E04	9.1E04	8.4E04	9.9E05	6.1E04	5.3E04	114.65
L. It.	0.03	6.1E04	9.3E04	8.4E04	9.4E05	5.9E04	5.2E04	112.52
F. It.	0.12	1.3E05	1.7E05	1.6E05	9.8E05	2.1E05	2.0E05	72.61
L. It.	0.12	1.7E05	1.9E05	1.8E05	1.1E06	2.0E05	2.3E05	72.95

TABLE VII. THE CONDITION NUMBER FOR THE POLAR COORDINATE SYSTEM FOR R = 3.03

It.	Load Factor	Measurement case						C.of Var. %
		1	2	3	4	5	6	
F. It.	0.03	8.4E06	7.5E06	8.7E06	1.0E07	7.8E06	7.8E06	7.97
L. It.	0.03	2.5E07	2.2E07	2.7E07	2.9E07	2.2E07	2.4E07	8.72
F. It.	0.12	9.3E06	8.1E06	9.5E06	1.1E07	8.5E06	8.5E06	8.56
L. It.	0.12	5.6E05	4.9E05	6.1E05	6.4E05	4.8E05	5.0E05	10.37

TABLE VIII. THE CONDITION NUMBER FOR THE RECTANGULAR COORDINATE SYSTEM FOR R = 3.03

It.	Load Factor	Measurement case						C.of Var. %
		1	2	3	4	5	6	
F. It.	0.03	1.6E04	1.6E04	1.7E04	1.6E04	1.1E04	1.1E04	16.09
L. It.	0.03	1.6E04	1.7E04	1.7E04	1.7E04	1.2E04	6.0E03	24.31
F. It.	0.12	2.5E04	2.4E04	2.3E04	1.9E04	2.5E04	2.5E04	7.09
L. It.	0.12	2.2E04	2.2E04	2.2E04	2.0E04	2.0E04	2.1E04	3.94

VIII. CONCLUSIONS

The features of the power system state estimation depends on many factors. In the paper. such factors as a coordinate system, in which a power system state vector is considered, stress of a power system, data redundancy, and also types of utilized measurements are taken into account. The results of the investigations show that especially stress of a power system and data redundancy have an essential impact on the WLS state estimation. In certain situation, also the mentioned coordinate system has significant impact on estimation calculations. Conditioning of a WLS estimation process is better for the rectangular coordinate system than for the polar coordinate system. The investigations show that also type of utilized measurements has impact on a power system state estimation. It can be observed that power-injection measurements have favorable impact on accuracy of estimation results. However. increasing a number of the power-injection measurements increases the value of a condition number which may lead to numerical problems.

REFERENCES

[1] A. Keyhani. and A. Abur. “Comparative study of polar and Cartesian co-ordinate algorithms for power-system state-estimation problems”. IEE Proc. on Gen..Trans. and Distr., vol. 132., pp. 132 - 138, May 1985.

[2] L. Holten, A. Gjelsvik, S. Aam, F. F. Wu, and W. E. Liu.: “Comparison of different methods for state estimation”. IEEE Trans. on Power Systems, vol. 3, pp. 1798 – 1806, May 1985.

[3] F. C. Schweppe, and J. Wildes, “Power System Static-State Estimation Part I: Exact Model”, IEEE Trans. on PAS, vol. 89, pp. 120-125, May 1985.

[4] F. C. Schweppe, J. Douglas, and B. Rom, “Power System Static-State Estimation Part II: Approximate model”, IEEE Trans. on PAS, vol. 89, pp. 125-130, May 1985.

[5] F. C. Schweppe, “Power System Static-State Estimation Part III: Implementation”, IEEE Trans. on PAS, vol. 89, pp. 130-135, May 1985.

[6] R. Jegatheesan, and K. Duraiswamy, “AC:Multi-terminal DC power system state estimation - a sequential approach”. Elec. Machines and Power Systems, vol. 12, pp. 27-42, January 1987.

[7] A. Nabavi-Niaki, and M.R. Iravani, “Steady-state and Dynamic Models of Unified Power Flow Controller (UPFC) for Power System Studies”, IEEE Trans. on Power Systems, vol. 11, pp. 1937-1943, May 1985.

[8] B. Xu, and A. Abur, “State Estimation of Systems With Embedded FACTS Devices”, IEEE Powertech Conf., Bologna, Italy, pp. 1-5, 2003.

[9] B. Xu, and A. Abur, “State Estimation of Systems With Embedded UPFCs Using the Interior Point Method”, IEEE Trans. on Power Systems, vol. 19, pp 1635-1641, August 2004.

[10] E. A. Zamora, and C.R. Fuerte-Esquivel, “Static state estimation of power systems containing series and shunt facts controllers”, 15th PSCC, Liege, pp. 1-6, 2005.

[11] C. Rakpenthai, S. Premrudepreechacharn, S. Uatrongjit, and N.R. Watson, “State Estimation of Power Systems with UPFC Using Interior Point WLAV Method”, 38th North American Power Symposium, pp: 411-415, 2006.

[12] S. K. Singh, and J. Dharma, “A Hopfield Neural Network based Approach for State Estimation of Power Systems Embedded with FACTS Devices”, IEEE Power India Conference, pp. 1-7, 2006.

[13] S. Guo-qiang, and W. Zhi-nong, “Power System State Estimation With Unified Power Flow Controller”, 3rd International Conference on Electric Utility Deregulation and Restructuring and Power Technologies, Nanjing, China, pp. 775-778, 2008.