Optimal Operation of a Distribution Company with integration of Reliability Considerations based on Generalized Benders Decomposition Method

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Abstract—This paper presents a comprehensive framework model of a distribution company with security and reliability consideration. Disco purchase its energy from its own DGs and in power market. Generalized Benders Decomposition method is implemented to solve Security constrained unit commitment. Only on/off binary variables are used as binary variables. Reliability indices, LOLP and EENS, are programmed with DELPHI language and linked with GAMS in order to satisfy reliability constraints in an iterative process. Reserve capacity is calculated as a way to improve the LOLP. Test results with an eight-bus system show the accuracy of the model and formulations.

***Index Terms*—Benders Decomposition, DISCO, EENS, Interruptible Loads, LOLP, SCUC, Spinning Reserve, Reliability Indices, Wind Farm.**

1. NOMENCLATURE

Constants

A_j	Coefficient of the piecewise linear production cost function of unit i.
a_i, b_i, c_i	Coefficients of quadratic production cost function of unit i.
NL_i	Number of the segments of the piecewise linear production cost function.
$\overline{P}_i$	Capacity of unit i
T	Number of periods of the time span.
$\underline{P}_i$	Minimum power output of unit i.
α_k, β_k	Coefficient factors of estimation customers
$P^{WH}_{Min}(k)$	Minimum power allowed from wholesaler
$P^{WH}_{Max}(k)$	Maximum power allowed from wholesaler
R^{Max}	Maximum spinning reserve.
RU_i	Ramp up limit of unit i.
SD^{DG}_i	Shutdown ramp rate of i^{th} DG.
S	Iteration number for saving EENS.
SR_{max}	Maximum Value of spinning reserve.
SU^{DG}_i	Startup ramp rate of i^{th} DG.
K	Number of periods of time span.

Variables

$p^{DG}_i(k)$	Power output of unit i.
$\overline{p}^{DG}_i(k)$	Maximum available power output of unit i period k.
$v_i(k)$	Binary Variables that is equal to 1 if unit i is online in period k and 0 otherwise.
$\delta_{l,i}(k)$	Power produced in block l of the piecewise linear production cost function of unit i in period k.
$CO^{DG}_i(t)$	Cost of i^{th} DG.
$\lambda^i_1(k)$	Dual variables of subproblem
$P_{IL}(t)$	Allowed interruptible loads.
$C_{IL}(t)$	Cost of interruptible loads.
$CS^{DG}_i(t)$	Startup cost of i^{th} DG.
$CD^{DG}_i(t)$	Shutdown cost of i^{th} DG.

Sets

I	Set of indexes of the generating units.
K	Set of indexes of the time periods.

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2. INTRODUCTION

IN general, the electrical energy sector over the past two decades or so has been affected by two important factors. The first factor is the advancement in generation technologies which has been evolving on a continuous basis, and newer and different energy transformation resources have been introduced to achieve high standards of energy provision. The second factor is the trend to liberate the energy sector from a monopolistic operating regime to a deregulated one, and to establish competitive markets for electricity [1].

Traditionally, a distribution company (Disco) purchases energy from wholesale market, at a high voltage level, and then transfers this energy to final customers. Nevertheless, the restructuring process of the energy sector has stimulated the introduction of new agents and products, and the unbundling of traditional Disco into technical and commercial tasks, including the provision of ancillary services [2].

A day-ahead energy acquisition model for a Disco in a pool market in the presence of financial bilateral contracts is presented in [3]. Both investor and utility-owned DG units are considered in the model and include IL options. An OPF model is used to arrive at the optimal set of energy schedules and decisions.

A multi-period energy acquisition model for a Disco with DG and IL options has been presented in [4]. A bi-level optimization formulation is developed wherein the upper subproblem maximizes the Disco’s revenue, while the lower subproblem addresses the independent system operator’s (ISO’s) market clearing by minimizing generation costs and compensation costs for IL. The model takes into consideration inter-temporal effects such as ramping.

In [5], a quantification of benefits from customer-owned back-up generators to Discos is carried out. An integration scheme for DGs in a pool-based market structure is proposed in [6] that encompasses both energy and capacity payment procedures. The problem of dispatch and control of DGs is formulated in [7] as a multi-agent system-based scheme, specifically for the purpose of voltage support.

A very important requirement that must be satisfied by the commitment schedule is the preservation of sufficient reserve to ensure adequate reliability levels, and proper characterization of it requires an examination of the relationship between the reserves and risk level proposed. Scheduling sufficient reserve capacity helps power systems overcome unscheduled generator outzages and major load forecasting errors without load shedding [8].

In [9], a method is described for determining the most economical generating unit commitment policy and loading schedule for a day's operation of an electric utility system while maintaining a desired level of system reliability.

Over the last four decades, numerous techniques and methods have been developed to calculate reserve-constrained unit commitment [10]. Most utilities have adopted deterministic criteria for the spinning reserve requirements. Their operating rules require the spinning reserve to the greater than the capacity of the largest online generator or a fraction of the load, or equal to some function of both of them [9].

As wind is an intermittent source of energy and cannot be predicted with high accuracy, reliability of wind farms compare to conventional power plants is lower. In [11], a reliability based unit commitment in restructured power systems with high penetration of wind farms is proposed. However the output of wind farm in [11] is the same in all hours.

In this paper, we propose a comprehensive operating framework of a distribution company. The probabilistic output of wind farm will be calculated in a 24 hour period. The linearized unit commitment’s equations are divided into two stages and are solved with benders decomposition method. In an iterative process, reliability indices, Loss of load probability (LOLP) as well as Expected Energy Not Served (EENS), will be calculated. If the Loss of Load Probability (LOLP) constraint is not satisfied in one iteration, the amount of spinning reserve will be increased in order to make the program add another unit in next iteration in order to improve the reliability. Since in this paper the reliability unit commitment is solved in a sequential procedure, the authors add a penalty term to total unit commitment cost and save the first five best results as it is depicted in Fig. 4.

The rest of this paper organizes as follows. Section II describes a modeling and mathematical formulation of a distribution company. Section III expresses the solution. Section IV discusses about the result of applying the proposed method on an eight test system. In Section V, conclusion of the study is represented. Finally the Benders Decomposition procedure is brought in Section VI.

3. MODELING AND MATHEMATICAL FORMULATION OF A DISTRIBUTION COMPANY

Fig.1 depicts the Model of the Disco in a day-ahead Electricity Market. This model consist several parts that are going to be described in details.

4. *Objective Function*

As it is stated in (1), the objective function consist three components. The first component expresses the cost function of Disco owned DG units. For sake of simplicity, the customer owned DGs are neglected in this model. The second component considers the cost function of purchased power from the market at the day-ahead market price which is imported via the sub-station transformer. The last element is the cost of interruptible load.

$$\min \sum_{t=1}^{T} \begin{bmatrix} \sum_{n=1}^{N} \left[CO_i^{DG}(t) + CS_i^{DG}(t) + CD_i^{DG}(t) \right] \\ + C\left(P^{WH}(t)\right) + C_{IL}(t).P_{IL}(t) \end{bmatrix} \tag{1}$$

5. *Operation cost of Disco owned DG:*

The I/O characteristic curve of Disco owned DGs is a quadratic and can be obtained from (2):

$$CO_i(t) = a_i.\left(P_i^{DG}\right)^2 + b_i.P_i^{DG} + c_i \tag{2}$$

As aforementioned above, all equations should be expressed in a linear fashion. Maintaining the convexity, (2) can be converted to multiple piecewise linear segments in order to approximate the non-linearity. The piecewise linear cost curve of DG i th is illustrated in Fig. 1.

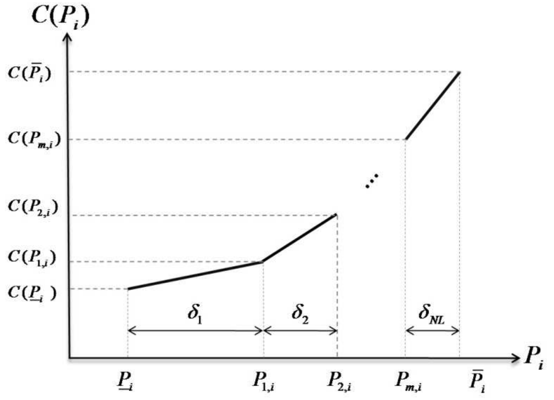

Fig. 1. Piecewise linear cost curve of unit i .

The analytic representation of this linear approximation by considering this fact that marginal costs are increasing is

$$CO_i^{DG}(t) = A_i + \sum_{l=1}^{NL} F_{l,i}.\delta_{l,i}, \quad \forall j \in J, \tag{3}$$

$$P_i(k) = \underline{P}_i(k).v(k) + \sum_{j=1}^{NL} \delta_{j,i}(k) \quad \forall j \in J \tag{4}$$

$$\delta_{1,i}(k) \le T_{1,i}(k) - \underline{P}_i(k), \quad \forall j \in J \tag{5}$$

$$\delta_{l,i}(k) \le T_{l,i}(k) - T_{l-1,i}(k), \quad \forall j \in J, \\ \forall l \in 2,\ldots, NL_i - 1 \tag{6}$$

$$\delta_{NL,i}(k) \le \overline{P}(k)_i - T_{l,i}(k), \quad \forall j \in J \tag{7}$$

$$\delta_{l,i}(k) \ge 0, \quad \forall j \in J, \forall l \in 1,\ldots, NL_i \tag{8}$$

6. *Purchasing from a wholesaler*

The input of the model includes customers' demand for 24 hours, wholesaler characteristic, and Disco owned DG units cost functions. Disco sells energy required by its customers at known price. However, it has to purchase some amount of energy at variable price of wholesaler. So Disco should estimate wholesaler's price which is given by (9) and (10):

$$P_{WH}(k) = \alpha_k.\rho_t(t) + \beta_k \tag{9}$$

$$P_{Min}^{WH}(K) \le P^{WH}(K) \le P_{Max}^{WH}(K) \tag{10}$$

Where $\alpha\, and\, \beta$ are obtained statistically from previous data and this fact that power cost has a linear relation with its price.

7. *Spinning Reserve*

$$0 \le \sum_{j=1}^{N} SR_j \le SR_{MAX} \tag{11}$$

8. *Constraints*

9. *Generation Limits and Ramping Constraints*

The generation limits of unit i for each period are set as follow (12) and (13):

$$\underline{P}_i^{DG}(k).v_i^{DG}(k) \le p_i^{DG}(k) \le \overline{p}_i^{DG}(k) \tag{12}$$

$$0 \le \overline{p}_i^{DG}(k) \le \overline{P}_i^{DG}(k).v_i^{DG}(k) \tag{13}$$

Constraints (12) bound the generation by the minimum power output and the maximum available power output of unit i in period k , $\overline{p}_i^{DG}$ which is a nonnegative variable bounded by (13), the unit capacity. It should be noted that while unit i is offline in period k , $v_i^{DG}(k) = 0$. So $p_i^{DG}(k)$ and $\overline{p}_i^{DG}(k)$ are equal to zero.

Variable $\overline{p}_i^{DG}(k)$ are also limited by ramp-up and startup ramp rates (14), as well as shutdown ramp rates constraints (15):

$$\begin{aligned} \overline{p}_i^{DG}(k) \le p_j(k-1) + \overline{P}_i\left[1 - v_i(k)\right] \\ + SU_i\left[v_i(k) - v_i(k-1)\right] \\ + RU_i.v_i(k-1) \end{aligned} \tag{14}$$

$$\begin{aligned} \overline{p}_i^{DG}(k) \le \overline{P}_i.v_i(k+1) + SD_i\left[v_i(k) - v_i(k+1)\right] \\ + RU_i.v_i(k-1), \forall t \in 1\ldots T-1 \end{aligned} \tag{15}$$

Moreover, ramp-down limits are imposed on the power output (16):

$$\begin{aligned} p_i^{DG}(k-1) - p_i^{DG}(k) \le SD_i.\left[v_i(k-1) - v_i(k)\right] \\ + \overline{P}_i.\left[1 - v_i(k-1)\right] \\ + RD.v_i(k) \end{aligned} \tag{16}$$

It should be noted that constraints (12)-(16) only include binary variables $v_i(k)$ and there is no need for extra variables as used in [12].

10. *Startup and shutdown Constraints*

As it is depicted in Fig. 2 [13], the startup cost function of a generator has an exponential nature that can be approximated in a discrete form as a stairwise function. A mixed integer linear formulation for stairwise startup cost [14] was proposed in (17) and (18):

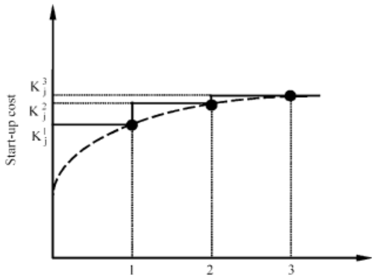

Fig. 2. Exponential, discrete, and stairwise startup cost functions.

$$c_j^u\left(k\right) = K_j^t\left[v_j\left(k\right) - \sum_{n=1}^{t} v_j\left(k-n\right)\right] \tag{17}$$

$$c_j^u\left(k\right) \geq 0 \tag{18}$$

As it is clear in (17) and (18), the linear startup cost functions only rely on the binary variables of committed units.

The shutdown cost which considers the fuel cost while the unit is going to be offline [14], is stated in (19) and (20):

$$c_j^d\left(k\right) = C_j\left[v_j\left(k-1\right) - v_j\left(k\right)\right] \tag{19}$$

$$c_j^d\left(k\right) \geq 0 \tag{20}$$

11. Linear Expression of Minimum Up and Down Time Constraints

Unlike [25] in which minimum up and down time constraints depend on binary variables associated with startup, shutdown, and on/off units, in this paper they only rely on committed units, $v_i(k)$. Minimum up time constraints are formulated as follows:

$$\sum_{k=1}^{G_i}\left[1-v_i\left(k\right)\right]=0 \tag{21}$$

$$\sum_{n=k}^{n=k+UT_i-1} v_i\left(n\right) \geq UT_i.\left[v_i\left(k\right)-v_i\left(k-1\right)\right] \tag{22}$$

$$\forall i \in I, \forall k \in G_i+1 \ldots T-UT_i+1$$

$$\sum_{n=k}^{T}\left\{v_i\left(n\right)-\left[v_i\left(k\right)-v_i\left(k-1\right)\right]\right\} \geq 0 \tag{23}$$

$$\forall k \in T-UT_i+2 \ldots T$$

Where G_i is the number of initial periods during which unit i must be online. G_i is mathematically expressed as $G_i=\min\left\{T,\left[UT_i-UT_i^0\right].V_i\left(0\right)\right\}$.

Constraints (21) are related to the initial status of the units as defined by G_i. Constraints (22) are used to subsequent periods to satisfy minimum up time constraint during all the possible sets of the consecutive periods of size UT_i. Constraints (23) model the final UT_i-1 periods in which if unit i is started up, it remains online unit until the end of the time span.

Analogously, minimum down time constraints are formulated as follows (24):

$$\sum_{k=1}^{L_i} v_i\left(k\right)=0, \quad \forall i \in I \tag{24}$$

$$\sum_{n=k}^{n=k+DT_i-1}\left[1-v_i\left(n\right)\right] \geq DT_i.\left[v_i\left(k-1\right)-v_i\left(k\right)\right] \tag{25}$$

$$\forall k \in L_i+1 \ldots T-DT_i+1$$

$$\sum_{n=k}^{T}\left\{1-v_i\left(n\right)-\left[v_i\left(k-1\right)-v_i\left(k\right)\right]\right\} \geq 0 \tag{26}$$

$$\forall k \in T-DT_i+2 \ldots T$$

Where L_i is the number of initial periods during which unit i must be offline. L_i is mathematically expressed as $L_i=\min\left\{T,\left[DT_i-S_i\left(0\right)\right].\left[1-V_i\left(0\right)\right]\right\}$.

12. Security Constraints

13. System Real Power Balance

$$\sum_{i \in I} P_i(k)+P_{WH}=D(k)+R(k), \quad \forall k \in K \tag{27}$$

14. Line Flow Constraints

$$-F_l^{Max}(k) \leq F_l(k) \leq F_l^{Max}(k), \quad \forall k \in K \tag{28}$$

15. Reliability Constraints

$$LOLP_t=\sum_{j=1}^{n} P_{R_j} LOSS_j \quad ,t \in T \tag{29}$$

$$LOLP \leq LOLP_{\max}$$

$$LOSS=\begin{cases}1, C_{Rj} \leq Load_t \\ 0, \quad Otherwise\end{cases} \tag{30}$$

If the LOLP constraint is not satisfied, the corresponding UC schedule for hour t will be rejected. However, a penalty should be calculated and added to total cost function while expected energy not supplied (EENS) is existed.

$$EENS=\sum_{j=1}^{n}\left[P_{Rj} \times Loss_j \times (Load_t - C_{Rj})\right]$$

$$EENS=\sum_{j=1}^{n}\left[P_{Rj} \times Loss_j \times (Load_t - C_{Rj})\right] \tag{31}$$

$$C_{ECOST}=VOLL*EENS \tag{32}$$

The VOLL is much higher than the cost of energy in developed countries depending on type of the customers and the period that customers are not supplied.

16. Probabilistic Model of a Wind Farm Output Power

The output power of a wind turbine generator is nonlinearly related to the wind speed. This model has three parameters namely cut in speed (V_{ci}), rated speed (V_r) and cut out speed (V_{co}) [15]. A schematic of wind farm output is depicted in Fig. 3. Output power of a WTG unit may be approximated by the following expression (35):

$$P_{Wind} = \begin{cases} 0 & V \le V_{ci} \\ \left(A + BV + CV^2\right) & V_{ci} < V \le V_r \\ P_r & V_r \le V < V_{co} \\ 0 & V \ge V_{co} \end{cases} \quad (33)$$

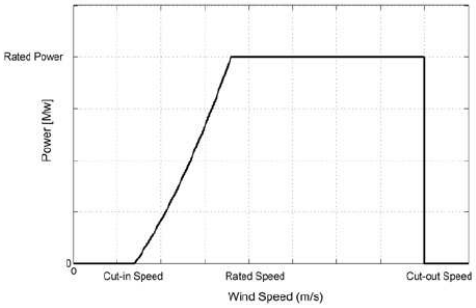

Fig. 3. Piecewise linear cost curve of unit i .

Where $A^{WIND}, B^{WIND}, and\ C^{WIND}$ are related to V_{ci} and V_r as it is stated in (36-38):

$$A^{WIND} = \frac{1}{(V_{ci} - V_r)^2}\left[V_{ci}(V_{ci} + V_r) - 4(V_{ci} \times V_r)\left[\frac{V_{ci} + V_r}{2V_{ci}}\right]^3\right] \quad (34)$$

$$B^{WIND} = \frac{1}{(V_{ci} - V_r)^2}\left[4(V_{ci} + V_r)\left[\frac{V_{ci} + V_r}{2V_r}\right]^3 - (3V_{ci} + V_r)\right] \quad (35)$$

$$C^{WIND} = \frac{1}{(V_{ci} - V_r)^2}\left[2 - 4\left[\frac{V_{ci} + V_r}{2V_r}\right]^3\right] \quad (36)$$

The output of a wind farm that has N_{WT} turbines is calculated as follows (39):

$$P_{WF}(t) = P_{Wind}(t) A_{Wind} \eta N_{WT} \quad (37)$$

In this paper a wind turbine rating power output was taken to be 5.0 MW. By having velocity of wind in eight different directions and their associated times of occurrence for six years in SIAHPOOSH, the capacity level and its probability for one wind turbine with considering forced outage rate (FOR) can be statistically derived as shown in Table I.

The expected average power of wind farm during a period of 24 hours is given in Table II.

17. SOLUTION METHOD

In this section, the solving procedure of minimizing total cost of Disco is proposed. Fig. 4 illustrates each step of solution method.

18. Security Constrained Unit Commitment Based on Benders Decomposition

Unit commitment in electric power systems is to optimize generating resources to supply system load while satisfying prevailing constraints, such as minimum on/off time, ramping up/down, minimum/maximum generating capacity, and fuel limit [16], [17].

TABLE I: Wind farm Combined Probability Distribution for each hour

Hour	**Probability of Occurrence**									
	50.0 MW	**44.0 MW**	**38.5 MW**	**33.0 MW**	**27.5 MW**	**22.0 MW**	**16.5 MW**	**11.0 MW**	**5.5 MW**	**0.0 MW**
1	0.818548	0.020327	0.015153	0.023166	0.018659	0.028228	0.016889	0.020518	0.022072	0.016439
2	0.807695	0.015938	0.019718	0.032179	0.025176	0.016384	0.022946	0.017832	0.021826	0.020304
3	0.785746	0.022605	0.026773	0.02827	0.018412	0.03013	0.024797	0.02426	0.014111	0.024897
4	0.764885	0.018981	0.029188	0.035746	0.016951	0.034347	0.025023	0.02228	0.027531	0.025067
5	0.746459	0.024967	0.037105	0.023652	0.020331	0.029074	0.029396	0.029236	0.029355	0.030423
6	0.746459	0.024967	0.037105	0.023652	0.020331	0.029074	0.029396	0.029236	0.029355	0.030423
7	0.725218	0.028932	0.035601	0.031239	0.028772	0.0234	0.029965	0.025509	0.036348	0.035015
8	0.706415	0.035451	0.027232	0.033117	0.036965	0.033558	0.025264	0.039471	0.032951	0.029577
9	0.688664	0.025491	0.039018	0.036956	0.049526	0.035399	0.028861	0.02972	0.031686	0.034679
10	0.687112	0.031644	0.038404	0.043661	0.037484	0.040602	0.02828	0.025189	0.038176	0.029450
11	0.691455	0.043882	0.041939	0.027913	0.033965	0.02351	0.02969	0.029581	0.031814	0.046259
12	0.667926	0.039839	0.030943	0.039759	0.039677	0.038537	0.036717	0.037996	0.046942	0.021664
13	0.680014	0.029563	0.021978	0.038813	0.040829	0.033277	0.033726	0.03578	0.03853	0.04749
14	0.694954	0.026239	0.033476	0.033375	0.026873	0.042714	0.033401	0.042306	0.033018	0.033644
15	0.681595	0.027552	0.026122	0.031481	0.034156	0.03965	0.046043	0.027637	0.031701	0.054061
16	0.669687	0.041023	0.030866	0.033923	0.036513	0.031527	0.046212	0.043174	0.032458	0.034618
17	0.709901	0.029333	0.034226	0.032659	0.038942	0.027333	0.029574	0.027794	0.039274	0.030963
18	0.7336701	0.028382	0.024214	0.033826	0.026897	0.028144	0.032793	0.030391	0.036497	0.025187
19	0.7378926	0.031653	0.020864	0.027788	0.033584	0.027692	0.038773	0.023567	0.033453	0.024733
20	0.7436435	0.027286	0.024931	0.025754	0.025641	0.030585	0.027332	0.031297	0.026752	0.036778
21	0.756614	0.033106	0.020549	0.034601	0.026051	0.019852	0.018945	0.031869	0.03031	0.028103
22	0.7854144	0.026957	0.027744	0.023654	0.024437	0.02271	0.021103	0.021168	0.019273	0.027539
23	0.793456	0.023663	0.025487	0.022799	0.025087	0.027165	0.021596	0.017834	0.022994	0.019918
24	0.8180212	0.018876	0.021151	0.016845	0.023261	0.021499	0.021484	0.018685	0.018704	0.021473

In this paper, a network model is also included to consider load flow limits which is based on a dc load flow. In general, the UC problem falls into the category of large scale and non-convex problems that are extremely difficult to solve in an accurate and efficient way. In light of the need of more efficient tools to support decisions for resource scheduling in the new competitive business environment, the Benders Decomposition method has attracted more attention in recent years.

The first part of the solving procedure is unit commitment problem with DC load flow constraints consideration. The operating problem addressed in this paper is formulated based on Benders Decomposition method. This approach decomposes UC into a master problem and a sub-problem. The master problem is an integer program and sub-problem is a linear program. The master problem applies the Mixed Integer Programming (MIP) method to solve UC and find proper on/off states of the units and the subproblem uses this solution to form appropriate cuts and adds them to the master problem for solving the next iteration of UC. The iterative process will continue until predefined gap is obtained and a converged optimal solution is found. In the proposed decomposition approach both sub-problem and master problem would have constraints. The objective function and constraints of the master problem are modified in a way that there exists no continuous variable in this problem, which helps to obtain the solution faster.

TABLE II: Expected Average Power Output of the wind farm

Hour	Capacity in (MW)	Hour	Capacity in (MW)
1	44.92956	13	40.44532
2	44.65466	14	41.16921
3	44.16840	15	40.38627
4	43.41394	16	40.71079
5	42.79737	17	41.86309
6	42.79737	18	42.41576
7	42.21640	19	42.62340
8	41.80897	20	42.51286
9	41.39469	21	43.20334
10	41.54502	22	44.16424
11	41.48085	23	44.41409
12	40.87347	24	44.87735

The Benders decomposition procedure followed in this paper includes the steps illustrated in the flowchart in Fig. 4.

Subproblem: The objective function of subproblem is as (40) in which the integer part is fixed.

$$\min \sum_{t=1}^{T} \sum_{n=1}^{N} \left[A_i + \sum_{l=1}^{NL} F_{l,i}.\delta_{l,i} + CD_n^{DG}\left(t\right) + CS_n^{DG}\left(t\right) \right] + \sum_{t=1}^{T} C_{WS}\left(t\right) \tag{38}$$

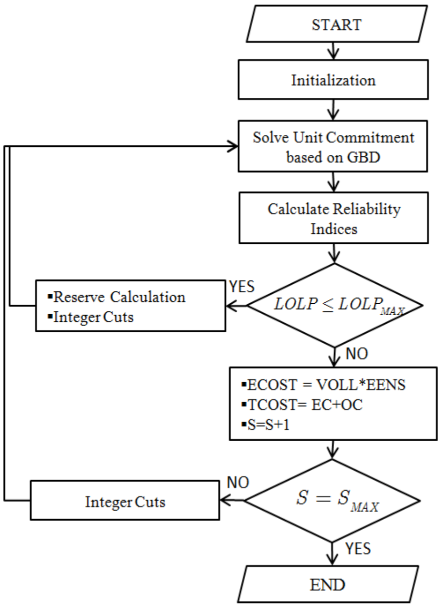

Fig 4 . Proposed Model of Disco with reliability and security constraints

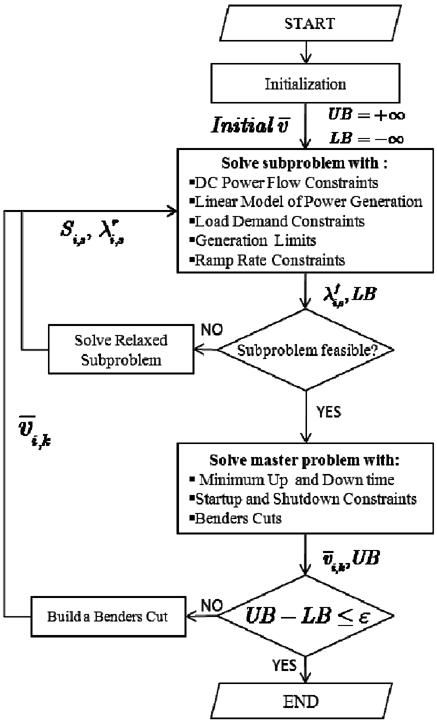

Fig. 5. General Procedure of Benders Decomposition Implementation

The subproblem will be solved subject to linear model of power generation (12), ramp-up and ramp-down constraints (13-15), power balance (27), and dc power flow equations (28).

Relaxed subproblem: If the subproblem is not feasible, the subproblem will be relaxed in order to make the subproblem feasible. The formulation of the relaxed subproblem is stated in (41-46):

$$\min \sum_{i=1}^{DG} \sum_{r=1}^{5} R_{i,r} \tag{39}$$

Subject to:

$$p_i^{DG}(k) - R_{i,1}^{DG} \ge \underline{P}_i^{DG}(k).\hat{v}_i^{DG}(k) \tag{40}$$

$$\overline{p}_i^{DG}(k) - R_{i,2}^{DG} \le \overline{P}_i^{DG}(k).\hat{v}_i^{DG}(k) \tag{41}$$

$$\begin{aligned}&\overline{p}_i^{DG}(k) - p_j(k-1) - R_{i,3}^{DG} \le +\overline{P}_i\left[1-\hat{v}_i^{DG}(k)\right] \\ &+ SU_i\left[\hat{v}_i^{DG}(k) - \hat{v}_i^{DG}(k-1)\right] + RU_i.\hat{v}_i^{DG}(k-1)\end{aligned} \tag{42}$$

$$\begin{aligned}&\overline{p}_i^{DG}(k) - R_{i,4}^{DG} \le \overline{P}_i.\hat{v}_i^{DG}(k+1) + SD_i\left[\hat{v}_i^{DG}(k) - \hat{v}_i^{DG}(k+1)\right] \\ &+RU_i.\hat{v}_i^{DG}(k-1)\end{aligned} \tag{43}$$

$$\begin{aligned}&p_i^{DG}(k-1) - p_i^{DG}(k) - R_{i,5}^{DG} \le SD_i.\left[\hat{v}_i^{DG}(k-1) - \hat{v}_i^{DG}(k)\right] \\ &+\overline{P}_i.\left[1-\hat{v}_i^{DG}(k-1)\right] + RD.\hat{v}_i^{DG}(k)\end{aligned} \tag{44}$$

After solving the relaxed subproblem and finding the duals variables, $\lambda_{i,r}^{R}$, we make a cut (47) and add it to the master problem.

$$\begin{aligned}&\lambda_{i,1}^{r}(k) * \left[\underline{P}_i^{DG}(k).v_i^{DG}(k) - p_i^{DG}(k)\right] \\ &+\lambda_{i,2}^{r}(k) * \left[\overline{p}_i^{DG}(k) - \overline{P}_i^{DG}(k).v_i^{DG}(k)\right] \\ &+\lambda_{i,3}^{r}(k) * \begin{bmatrix}\overline{p}_i^{DG}(k) - p_j(k-1) - \overline{P}_i\left[1-v_i(k)\right] \\ -SU_i\left[v_i(k) - v_i(k-1)\right] - RU_i.v_i(k-1)\end{bmatrix} \\ &+\lambda_{i,4}^{r}(k) * \begin{bmatrix}\overline{p}_i^{DG}(k) - \overline{P}_i.v_i(k+1) - RU_i.v_i(k-1) \\ -SD_i\left[v_i(k) - v_i(k+1)\right]\end{bmatrix} \\ &+\lambda_{i,5}^{r}(k) * \begin{bmatrix}p_i^{DG}(k-1) - p_i^{DG}(k) - \overline{P}_i.\left[1-v_i(k-1)\right] \\ -SD_i.\left[v_i(k-1) - v_i(k)\right]\end{bmatrix} \le 0\end{aligned} \tag{45}$$

Master Problem: The objective function of the master problem is as (40) in which the continuous part is fixed. Constraints regarding to the master problem are startup and shutdown constraints (17-20) and minimum up time (21-23) and minimum down time constraints (24-26).

A benders cut (48) is added to the master problem while the master problem does not converged:

$$\begin{aligned}&Cut = \lambda_1^{i}(k) * \left[\underline{P}_i^{DG}(k).v_i^{DG}(k) - p_i^{DG}(k)\right] \\ &+\lambda_2^{i}(k) * \left[\overline{p}_i^{DG}(k) - \overline{P}_i^{DG}(k).v_i^{DG}(k)\right] \\ &+\lambda_3^{i}(k) * \begin{bmatrix}\overline{p}_i^{DG}(k) - p_j(k-1) - \overline{P}_i\left[1-v_i(k)\right] \\ -SU_i\left[v_i(k) - v_i(k-1)\right] - RU_i.v_i(k-1)\end{bmatrix} \\ &+\lambda_4^{i}(k) * \begin{bmatrix}\overline{p}_i^{DG}(k) - \overline{P}_i.v_i(k+1) - RU_i.v_i(k-1) \\ -SD_i\left[v_i(k) - v_i(k+1)\right]\end{bmatrix} \\ &+\lambda_5^{i}(k) * \begin{bmatrix}p_i^{DG}(k-1) - p_i^{DG}(k) - \overline{P}_i.\left[1-v_i(k-1)\right] \\ -SD_i.\left[v_i(k-1) - v_i(k)\right]\end{bmatrix}\end{aligned} \tag{46}$$

The master problem will be solved by either (47) or (48) and the optimization problem will be terminated while $UB - LB \le \varepsilon$.

19. Calculation of Reliability Indices

To apply reliability constraints, the LOLP and EENS reliability indexes are incorporated in the formulation of the problem. The LOLP reliability constraint is given by (32):

Since this paper is programmed in GAMS environment, calculating LOLP implicitly in GAMS bring lots of computational complexities [10]. In this paper, LOLP and EENS are computed explicitly have programmed in DELPHI environment. So LOLP and EENS are computed by the result of unit commitment which is written in GAMS, and the results are sent back to GAMS environment. Fig. 6 illustrates this procedure:

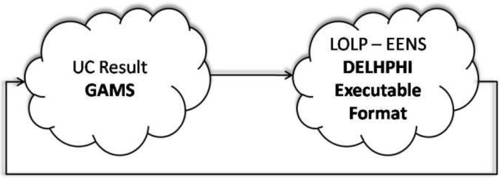

Fig. 6. Proposed Model of Disco with reliability and security constraints

This iterative process continue until (32) is satisfied for all 24 hours. If hour k (32) is not satisfied, it means that LOLP is not suitable. In another word, the committed generators are not enough. So integer cuts and reserve calculation makes the program to choose combination of generators. As it is stated in (39) and (40), an increase in reserve capacity makes the program choose another set of generators:

$$reserve(k) = \left(\sum_{i=1}^{N}\left((P_i^{MAX}(k) * v_i(k)\right) - \sum_{i=1}^{N}\left(P_i(k) * v_i(k)\right)\right) + \varepsilon \tag{47}$$

$$0 \le reserve(k) \le R^{Max}(k) \tag{48}$$

If LOLP is satisfied for whole period, in the next step penalty factor which is VOLL[1] multiplied by EENS which is called reliability cost is computed and then is added to the total cost of the problem (35)-(36).

$$Cost_{Reliability} = EENS * VOLL \tag{49}$$

$$Total\,Cost = Cost_{Operation} + Cost_{Reliability} \tag{50}$$

Since this method is sub optimum, the first S^{th} results are saved and the minimum answer is selected as the optimum result.

20. CASE STUDY

An eight bus test system is depicted in Fig. 7[18]. There are five thermal units and one hydro unit, ten transmission lines, and a phase shifter is installed on line 5. The characteristics of generators, buses, and transmission lines are listed in Tables III -V, respectively. The 24-hour system load is presented in Table IV.

Fig. 7. A eight bus test system

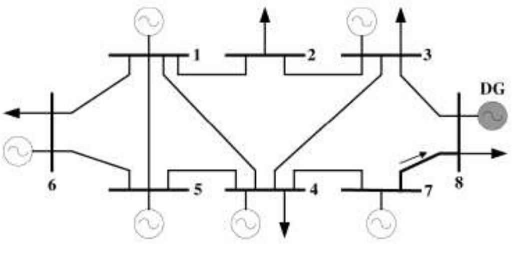

TABLE III: PARAMETERS OF HYDRO UNIT

Unit	a_1 $(\$/MW^2)$	b $(\$/MW)$	c $(\$)$	$Start\,UP$ $Cost(\$)$	$Flow$
G6	7	-0.144	8.55	-6.45	0.5

---

[1] Value of the loss load

Unit	$Min\,ON$	$Min\,OFF$	*Initial Value*	$V_{\min}$	$V_{\max}$
G6	1	1	200	150	250

TABLE IV: PARAMETERS OF THERMAL UNITS

Unit	G1	G2	G3	G4	G5
Bus No.	1	2	3	4	5
$a_1(\$/MW^2)$	0.0143	0.0085	0.0081	0.0046	0.0014
$b(\$/MW)$	27.89	19	18.1	12.69	9.66
$c(\$)$	30	27	27	25	20
$P_{\min}(MW)$	5	10	10	20	70
$P_{\max}(MW)$	50	50	50	70	120
$Start\,UP\,Cost(\$)$	50	50	50	50	500
MSR	0.8	0.8	1.9	1.9	2.5
$Ramp\,UP$	0.83	0.83	1.25	1.66	2.95
$Min\,ON$	1	1	5	8	8
$Min\,OFF$	1	1	5	8	8
Initial State	-1	-1	-5	8	10

TABLE V: TRANSMISSION LINES AND PHASE SHIFTER PARAMETERS

Line No	From Bus	To Bus	X (pu)	Line Limit (MW)	Phase Shifting angel
1	1	2	0.03	200	0
2	1	4	0.001	200	0
3	1	7	0.03	200	0
4	2	3	0.03	200	0
5	5	2	0.001	200	0
6	4	5	0.03	200	0
7	5	6	0.03	200	0
8	6	8	0.03	200	0
9	7	4	0.03	200	0
10	8	3	0.03	200	0

TABLE VI: LOAD DEMAND

Hour	Load (MW)	Hour	Load (MW)	Hour	Load (MW)
1	265.5333	9	428.6875	17	424.8250
2	256.6466	10	420.1138	18	428.6875
3	261.0707	11	411.54	19	424.8250
4	287.6536	12	407.1159	20	398.2807
5	318.6348	13	398.2807	21	354.0273
6	376.1476	14	389.4197	22	309.7867
7	411.54	15	398.2807	23	354.0273
8	420.1138	16	407.1159	24	309.7867

The proposed method was compared with [18].

In this section two different scenarios are presented.

1) In the first scenario, reliability indices are not considered and only unit commitments with network constraints are considered. The generation dispatch is presented in Table V. As it is extracted from Table V, unit 5 and 1 is the cheapest and the most expensive unit, respectively.

2) In the second scenario, reliability constraints are also added to the system constraints. Table VI states the LOLP of the optimum result. Table VII includes the first 10th EENS result. As it is extracted from table VII, the first EENS has the minimum EENS among all first 10th EENS.

TABLE VII: Best LOLP Result

Hour	1	2	3	4	5	6	7	8
LOLP	0.019	0.0.3	0.011	0.14	0.16	0.04	0.044	0.023
Hour	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
LOLP	0.04	0.044	0.023	0.013	0.04	0.012	0.05	0.047
Hour	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
LOLP	0.014	0.034	0.01	0.04	0.05	0.049	0.034	0.013

TABLE VII: first 5th EENS

No.	1	2	3	4	5
EENS	21.31	23.11	24.65	25.31	25.52

TABLE VII: Generation Dispatch with security and reliability Calculation without Reliability Calculation

UNIT	G1	G2	G3	G4	G5	G6
1	0	0	0	65.5	200	0
2	0	0	0	46.6	200	0
3	0	0	0	61.1	200	0
4	0	10	0	77.6	200	0
5	0	17.1	10	91.6	200	0
6	0	50	40.9	85.2	200	0
7	0	50	70	70.9	200	20.6
8	0	50	70	67.7	200	46.3
9	0	50	50.6	67.6	200	48.7
10	0	50	65.6	71.2	200	46.3
11	0	50	66.1	72.7	200	22.1
12	0	50	53.6	76.2	200	14.5
13	0	50	50.6	79.8	200	6.1
14	0	50	65.6	76.1	200	6.1
15	0	50	66.1	79.8	200	6.1
16	0	50	70	76.1	200	14.5
17	0	50	69.4	72.6	200	46.3
18	0	50	70	65.7	200	48.7
19	0	50	69.4	64.2	200	46.3
20	0	50	50	65.7	200	0
21	0	49.8	10	76.4	200	0
22	0	10	10	94.2	200	0
23	0	49.8	10	89.8	200	0
24	0	15.4	0	94.2	200	0

Table VIII states the generation dispatch with security as well as reliability constraints. As it is clear in Table VIII, in some hours one unit is added to committed units in order to improve the reliability that makes the system more reliable. However the total cost will increase. Table IX compares these two scenarios. The total cost includes total generation cost and reliability cost of the system.

TABLE VI: Total Cost of two Scenarios

Total Cost without Reliability Constraints	$109,936.00
Total Cost without Reliability Constraints	$115143.10

21. CONCLUSION

This paper presented an operating model of Distribution Company with security as well as reliability consideration. The proposed algorithm was applied to the an eight bus test system and the results were presented and discussed. A step by step procedure was illustrated to clarify the applicability of proposed methodology. The proposed algorithm was applied to an eight bus test system and the results were presented and discussed. A step by step procedure was illustrated to clarify the applicability of proposed methodology. In unit commitment formulation only one set of binary variables, on/off of the generators, were used. The Generalized Benders Decomposition was used to solve the unit commitment problem. LOLP and EENS were programmed in Delphi environment and linked with GAMS. The model was tested on an eight bus test system and two scenarios are done.

IX: Generation Dispatch with security and reliability Calculation with Reliability Calculation

UNIT	G1	G2	G3	G4	G5	G6
1	0	0	0	65.5	200	0
2	0	0	0	46.6	200	0
3	0	0	0	61.1	200	0
4	0	10	0	77.6	200	0
5	0	17.1	10	91.6	200	0
6	0	50	40.9	85.2	200	0
7	0	50	70	70.9	200	20.6
8	0	50	70	67.7	200	46.3
9	5	50	50.6	67.6	200	43.7
10	5	50	65.6	71.2	200	41.3
11	0	50	66.1	72.7	200	22.1
12	0	50	53.6	76.2	200	14.5
13	0	50	50.6	79.8	200	6.1
14	0	50	65.6	76.1	200	6.1
15	0	50	66.1	79.8	200	6.1
16	5	50	70	76.1	200	9.5
17	5	50	69.4	72.6	200	41.3
18	5	50	70	65.7	200	43.7
19	5	50	69.4	64.2	200	41.3
20	0	50	50	65.7	200	0
21	0	49.8	10	76.4	200	0
22	0	10	10	94.2	200	0
23	0	49.8	10	89.8	200	0
24	0	15.4	0	94.2	200	0

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