Optimal interconnecting wind generation into Polish Power System

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***Abstract*— The main objective of a Transmission System Operator is to operate the system in a safe, secure and efficient manner. Hence, the transmission lines and transformers should not be overloaded after interconnecting wind farms into transmission network. Wind generation results in decreasing conventional generation. On the other hand, the technical minimum of a thermal unit in power stations must not be violated. Such a task can be formulated as the minimization of a linear goal function subjected to linear equality (generation and demand balance) and inequality constraints (permissible branch flows and thermal generation minimum). The idea of a paper is to formulate and solve the problem as a linear programming task. Then the Matlab Optimization Tools is used to solve the linear optimization task. The mathematical background and the small example power system are presented in the paper.**

Keywords-distributed generation, wind generation, optimization

I. INTRODUCTION

Wind generation impacts on power system regulation, reliability and efficiency. These impacts can not only be positive, but sometimes negative. The security of power system is the main goal of planning the adequate quantity of thermal, water and wind generation. In order to manage disturbances the power system should fulfil the requirements of enough reserves in power plants and in the transmission grid and the power transfers should be kept in the allowed limits.

Wind generation affects power flow in the transmission network and may even change the branch power direction. Improper location of wind farm can effect bottleneck situations.

The main objective of a Transmission System Operator is to operate the system in a safe, secure and efficient manner. Hence, the transmission lines and transformers should not be overloaded after interconnecting wind farms into transmission network. Wind generation results in decreasing conventional generation, Fig. 1. On the other hand, the technical minimum of a thermal unit in power stations must not be violated. Such a task can be formulated as the minimization of a linear goal function subjected to linear equality (generation and demand balance) and inequality constraints (permissible branch flows and the minimal power of thermal units). Next, the contingency N-1 should be checked in the optimal state.

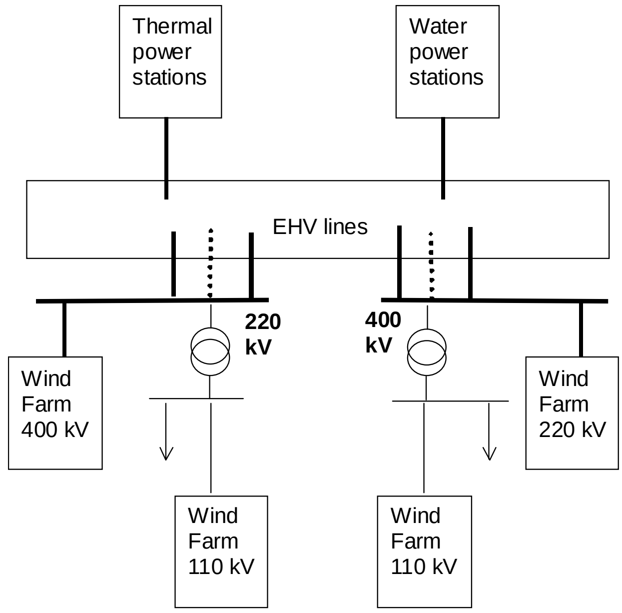

Figure 1. The interconnection of wind farms to the transmission network

The idea of a paper is to formulate and solve the problem as a linear programming task. Then the Matlab Optimization Tools is used to solve the linear optimization task. The mathematical background and example computation are presented in the paper.

II. MATHEMATICAL BACKGROUND

It is analysed whether certain technical conditions are met after interconnecting wind farms to the transmission grid buses. It means that for a certain topology, some branch powers should be within a range in order to be acceptable. The general formulation of the problem is as follows

$$\min f(x) \tag{1}$$

subject to

$$\mathbf{A_{eq}x = b_{eq}} \quad (1a)$$

$$\mathbf{Ax \leq b} \quad (1b)$$

$$\mathbf{x \geq x_{min}} \quad (1c)$$

$$\mathbf{x \leq x_{max}} \quad (1d)$$

Symbol x means the vector of buses with a wind farm and the goal function is linear.

Simplifying assumptions

1. In planning there are considered states with safe stability margins – that means with small angles differences across branches, what means:

$$\sin(\delta_i - \delta_j) \approx \delta_i - \delta_j \quad (2)$$

$$\cos(\delta_i - \delta_j) \approx 1 \quad (3)$$

2. Voltage drops in lines and transformers are low, what means that node voltages values are approximately equal to their rated values:

$$U_i \approx U_N \quad (4)$$

In consequence of the simplifying assumptions the power flow is described with a liner matrix equation:

$$\mathbf{P} = \mathbf{B}\boldsymbol{\delta} \quad (5a)$$

or

$$\boldsymbol{\delta} = \mathbf{B^{-1}P = XP} \quad (5b)$$

where:

P – node active powers vector,

B – node susceptance matrix,

X – node reactance matrix,

$\boldsymbol{\delta}$ – node voltage angles vector.

w = n –1- independent nodes number,

n – slack node number.

Node powers vector is the difference of powers generated and received at network nodes:

$$\mathbf{P = P_g - P_d} \quad (6a)$$

where:

$\mathbf{P_g}$ – generated node powers vector,

$\mathbf{P_d}$ – received node powers vector /demanded/.

Branch powers can be directly calculated from node powers:

$$\mathbf{P_b} = \mathbf{diag(b_{branch})C}\boldsymbol{\delta} = \mathbf{C_b\ X\ P = H\ P} \quad (6b)$$

where:

$\boldsymbol{\delta} = \mathbf{XP}$ - node angles vector,

$\mathbf{H = C_b\ X}$ - power transfer matrix,

$\mathbf{C_b = diag(b_{branch})\ C}$ – branch-node connections admittance matrix,

$\mathbf{b_{branch}}$ - branch susceptances vector,

C - zero-ones branch-node connections matrix.

III. EQUALITY CONSTRAINTS

Equality constraints arise mainly from power balance equations:

$$\mathbf{e^T(P_g - P_d)} + P_n = 0 \quad (7)$$

where:

$\mathbf{e^T}$ = [1 1 ... 1] - ones vector of the size of w=n-1,

P_n - the active generation at the slack bus.

The equation (7) can be transformed in the formula, in which on the left side we have the optimized variables vector and on the right side - the given values:

$$\mathbf{e^T P_g} = \mathbf{e^T P_d} - P_n \quad (7a)$$

Taking into account that the new sources can be connected only to some nodes, the node power vector can be divided into two sub-vectors:

$$\mathbf{P} = \begin{bmatrix} \mathbf{P_x} \\ \mathbf{P_y} \end{bmatrix} = \begin{bmatrix} \mathbf{P_{gx}} - \mathbf{P_{dx}} \\ \mathbf{P_{gy}} - \mathbf{P_{dy}} \end{bmatrix} \quad (8)$$

where:

x - index for node with wind farm,

y - index for node with other generation.

In consequence, the equation (7) assumes the following shape:

$$\begin{bmatrix} \mathbf{e_x^T} & \mathbf{e_y^T} \end{bmatrix} \begin{bmatrix} \mathbf{P}_{gx} \\ \mathbf{P}_{gy} \end{bmatrix} = \mathbf{e^T P_d} - P_n \quad (9a)$$

$$\mathbf{e_x^T P_{gx}} = -\mathbf{e_y^T P_{gy}} + \mathbf{e^T P_d} - P_n \quad (9b)$$

To the equality constraints also the constant power international transfer equations have to be included:

$$\mathbf{P_{bwym}} = \begin{bmatrix} \mathbf{H_{xwym}} & \mathbf{H_{ywym}} \end{bmatrix} \begin{bmatrix} \mathbf{P_x} \\ \mathbf{P_y} \end{bmatrix} \quad (10)$$

$$\mathbf{P_{bwym}} = \mathbf{H_{xwym} P_x} + \mathbf{H_{ywym} P_y} \quad (10a)$$

$$\mathbf{P_{bwym}} = \mathbf{H_{xwym} P_{gx}} - \mathbf{H_{xwym} P_{dx}} + \mathbf{H_{ywym} P_y} \quad (10b)$$

$$\mathbf{H_{xwym} P_{gx}} = \mathbf{P_{bwym}} + \mathbf{H_{xwym} P_{dx}} - \mathbf{H_{ywym} P_y} \quad (10c)$$

Power balance equations and international power exchange equations written together create the equality constraints in the optimization task:

$$\begin{bmatrix} \mathbf{e_x^T} \\ \mathbf{H_{xwym}} \end{bmatrix} \mathbf{P_{gx}} = \begin{bmatrix} -\mathbf{e_y^T P_{gy}} + \mathbf{e^T P_d} - P_n \\ \mathbf{P_{bwym}} + \mathbf{H_{xwym} P_{dx}} - \mathbf{H_{ywym} P_y} \end{bmatrix} \quad (11)$$

The equality constraints can be written in a form suitable for Matlab calculations:

$$\mathbf{A_{eq} x} = \mathbf{b_{eq}} \quad (12)$$

where:

$\mathbf{x} = \mathbf{P_{gx}}$ - new node optimized generations vector,

$\mathbf{A_{eq}} = \begin{bmatrix} \mathbf{e_x^T} \\ \mathbf{H_{xwym}} \end{bmatrix}$ - equality constraints matrix,

$\mathbf{b_{eq}} = \begin{bmatrix} -\mathbf{e_y^T P_{gy}} + \mathbf{e^T P_d} - P_n \\ \mathbf{P_{bwym}} + \mathbf{H_{xwym} P_{dx}} - \mathbf{H_{ywym} P_y} \end{bmatrix}$ - right-side equality constraints vector.

IV. BRANCH INEQUALITY CONSTRAINTS

Inequality constraints result from branch allowable powers:

$$\mathbf{H(P_g - P_d)} \le \mathbf{S_{max}} \quad (13)$$

where:

$S_{max} = U_N I_{max}$ - a maximal branch flow,

I_{max} - a maximal branch current.

However, it has to be taken into account, that change in generated powers can lead to change in power flow direction, therefore a change in power sign. For this reason the equation (13) has to be supplemented with an additional inequality:

$$-\mathbf{H(P_g - P_d)} \le \mathbf{S_{max}} \quad (14)$$

Both inequalities can be written together in matrix form:

$$\mathbf{HP_g} \le \mathbf{S_{max}} + \mathbf{HP_d} \quad (15)$$

$$-\mathbf{HP_g} \le \mathbf{S_{max}} - \mathbf{HP_d} \quad (16)$$

$$\begin{bmatrix} \mathbf{H} \\ -\mathbf{H} \end{bmatrix} \mathbf{P_g} \le \begin{bmatrix} \mathbf{S_{max}} + \mathbf{HP_d} \\ \mathbf{S_{max}} - \mathbf{HP_d} \end{bmatrix} \quad (17)$$

Inequality (17) can be written in a shorter form:

$$\mathbf{AP_g} \le \mathbf{b} \quad (18)$$

where:

$\mathbf{A} = \begin{bmatrix} \mathbf{H} \\ -\mathbf{H} \end{bmatrix}$ - inequality constraints matrix,

$\mathbf{b} = \begin{bmatrix} \mathbf{S_{max}} + \mathbf{HP_d} \\ \mathbf{S_{max}} - \mathbf{HP_d} \end{bmatrix}$ - right-side equality constraints vector.

Taking into account the division of generated powers vector on to two sub-vectors, inequality (18) can be rewritten into following form:

$$\begin{bmatrix}\mathbf{A}_x & \mathbf{A}_y\end{bmatrix}\begin{bmatrix}\mathbf{P_{gx}} \\ \mathbf{P_{gy}}\end{bmatrix} \le \mathbf{b} \tag{19}$$

$$\mathbf{A_x P_{gx}} + \mathbf{A_y P_{gy}} \le \mathbf{b} \tag{19a}$$

$$\mathbf{A_x P_{gx}} \le \mathbf{b} - \mathbf{A_y P_{gy}} \tag{19b}$$

Inequality (19b) can be rewritten into form suitable for Matlab calculations:

$$\mathbf{A}_{\mathbf{ineq}}\mathbf{x} = \mathbf{b}_{\mathbf{ineq}} \tag{20}$$

where:

$\mathbf{x} = \mathbf{P_{gx}}$ - new node optimized generations vector,

$\mathbf{A_{ineq}} = \mathbf{A_x}$ - inequality constraints matrix,

$\mathbf{b_{ineq}} = \mathbf{b} - \mathbf{A_y P_{gy}}$ - right-side equality constraints vector.

V. GENERATION INEQUALITY CONSTRAINTS

Generation constraints arise from power unit's energy generation technology. That are constraints creating technical minimum for power industry power generation:

$$\mathbf{P_{gx}} \le \mathbf{P_{gxmax}} \tag{27}$$

$$\mathbf{P_{gx}} \ge \mathbf{P_{gxmin}} \tag{28}$$

Power system generation technical minimum is defined as a sum of minimal powers of thermal power units.

$$P_{g\,min\,SEE} = \mathbf{e_y^T P_{gymin}} \tag{29}$$

VI. CONSIDERATION OF NODE EQUIVALENT LOAD POWER INACCURACY

Indeterminacy of equivalent load powers feed from 110 kV buses of NN/110 kV transformers can be considered as random variables with known expected value and known variations.

Most pessimistic description of random variables is the multidimensional rectangular distribution of independent variables, giving equal probability for each value from examined variation interval:

$$\mathbf{P}_{\mathrm{d}\mathbf{m}\mathrm{in}} \le \mathbf{P}_{\mathrm{d}} \le \mathbf{P}_{\mathrm{d}\mathbf{max}} \tag{30}$$

The minimal and maximal value responds to given percent modelling accuracy eg. 5%.

The random variable expected value with rectangular probability distribution vector results from equation:

$$\mathbf{m_d} = \mathrm{E}(\mathbf{P}_{\mathrm{d}}) = (\mathbf{P}_{\mathrm{d}\mathbf{m}\mathrm{ax}} - \mathbf{P}_{\mathrm{d}\mathbf{m}\mathrm{in}})/2 \tag{31}$$

Received power variation at the i-th node equals:

$$\mathrm{var}(P_{di}) = (P_{dmaxi} - P_{dmini})^2/12 \tag{32}$$

In case of all independent nodes the variances create a diagonal covariance matrix:

$$\mathbf{M_d} = \mathbf{diag}(\mathrm{var}(P_{di})) \tag{33}$$

Random received power changes at nodes cause random changes in branch flows, according to equality (5), which can now be rewritten with received powers vector distinguishing:

$$\mathbf{P_b} = \mathbf{H}(\mathbf{P_g} - \mathbf{P_d}) = \mathbf{HP_g} - \mathbf{HP_d} \tag{34}$$

In consequence the branch powers vector $\mathbf{P_b}$ should be treated as a linear function of random received powers $\mathbf{P_d}$. In accordance with linear transformation properties of random variables, the new variable has a multidimensional normal probability distribution with a following expected value vector:

$$\mathbf{m_b} = \mathbf{HP_g} - \mathbf{Hm_d} \tag{35}$$

The new random variable variances are diagonal elements of a following covariance matrix:

$$\mathbf{M_b} = \mathbf{HM_dH^T} \tag{34}$$

Branch power in arbitrary chosen branch subjects to normal probability distribution with following probability density:

$$\mathrm{N}(m,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(z-m)^2}{2\sigma^2}\right] \tag{35}$$

where:

m – the expected value of power in arbitrary chosen branch, equal to corresponding element of $\mathbf{m_b}$ vector, σ – the power standard deviation in random branch, equal square of its power variance,

σ^2 – the power variation in random branch, equal to corresponding $\mathbf{M_b}$ covariance matrix diagonal element,

z – a random power in arbitrary chosen branch.

Based on probability distribution, there can be determined the interval probability in which the branch power value will be found. According to 3-sigma rule, the largest and smallest node power value can be estimated with 0.997 probability, up to the pattern:

$$z_{min} = m - 3\sigma \tag{36}$$

$$z_{max} = m + 3\sigma \tag{37}$$

From the overload point of view, the most significant is the largest value. An increment in branch power by 3 sigma in proportion to expected value let's further treat the task as deterministic, but with reduced branch constraint interval:

$$S_{max} = S_{max} - 3\sigma \text{ with probability of } 0.997 \tag{38}$$

It is also possible to consider the smaller acceptable power exceed probability values, answering the 2- or 1-sigma deviations:

$$S_{max} = S_{max} - 2\sigma \text{ with probability of } 0.954 \tag{38a}$$

$$S_{max} = S_{max} - \sigma \text{ with probability of } 0.683 \tag{38b}$$

VII. OBJECTIVE FUNCTION

There are several possible objective function forms:

1. **Maximal total wind generation.** Its main disadvantage results from the fact, that this value is constant and independent from individual wind turbines powers.
2. **Minimal total cost of wind generation.** The best possible function form, but it requires the knowledge on node powers cost. Objective function from p.1 responds to this objective function with equal node power costs.
3. **Maximal total lines and transformers loading** (minimal sum of branch power margins). It loads the weakly loaded branches, but can cause lowering of transmission safety on heavy loaded lines before optimization.
4. **Minimal total lines and transformers loading** (maximal sum of branch power margins). It unloads heavy loaded lines. These operation consequences have to be considered.
5. **Exchange power deviations sum minimization.** It responds to secondary exchange power and frequency regulation.
6. **Total transmission losses minimization**. It changes the task from linear programming to quadratic programming, what causes increase in calculation time.

VIII. CALCULATION EXAMPLE

In order to illustrate the considerations, wind farm power optimization in 4-node system has been made, Fig. 2. The wind farms are connected to nodes 1 and 2, covering the power demand at node 3, totalling P_3=20.

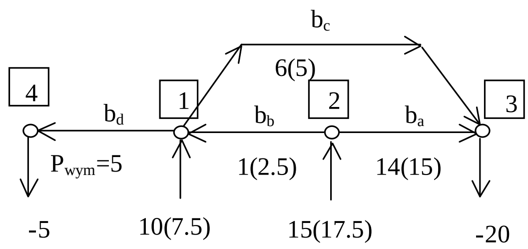

Figure 2. Power flow resulting from optimal wind farms power selection. In brackets the farm powers and power flow fulfilling the system constraints have been given.

The nominal voltage is U_N = 400 kV and the base power S_b = 100 MVA. Hence the base impedanc equals

$$Z_b = \frac{U_N^2}{S_b} = \frac{400^2}{100} = 1600\ \Omega$$

Between node 4 and 1 flows constant exchange power P_{wym}=5. Allowable branches powers equal: S_{amax}=20, S_{bmax}=5, S_{cmax}=5, S_{dmax}=10. Wind farms arbitrary powers P_1=10, P_2=15 ensure exchange power P_{wym}=5, but they don't meet the constraints.

Relevant matrices and vectors in per unit are as follows:

$$\mathbf{z_{branch}} = \begin{bmatrix} \underline{z}_a \\ \underline{z}_b \\ \underline{z}_c \\ \underline{z}_d \end{bmatrix} = \begin{bmatrix} 0.0012 + j0.0124 \\ 0.0048 + j0.0240 \\ 0.0025 + j0.0248 \\ 0.0015 + 0.0123 \end{bmatrix}$$ - branch impedances,

$$\mathbf{y_{branch}} = \begin{bmatrix} \underline{y}_a \\ \underline{y}_b \\ \underline{y}_c \\ \underline{y}_d \end{bmatrix} = \begin{bmatrix} 8 - j80 \\ 8 - j40 \\ 4 - j40 \\ 10 - j80 \end{bmatrix}$$ - branch admitances,

$$\mathbf{b}_{\mathbf{branch}} = \begin{bmatrix} b_a \\ b_b \\ b_c \\ b_d \end{bmatrix} = \begin{bmatrix} -80 \\ -40 \\ -40 \\ -80 \end{bmatrix}$$ - branch susceptances,

$$\mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ -20 \end{bmatrix}$$ - node powers,

$$\mathbf{B} = \begin{bmatrix} 160 & -40 & -40 \\ -40 & 120 & -80 \\ -40 & -80 & 120 \end{bmatrix}$$ - node susceptance matrix,

$$\mathbf{X} = \mathbf{B}^{-1} = \begin{bmatrix} 0.0125 & 0.0125 & 0.0125 \\ 0.0125 & 0.0275 & 0.0225 \\ 0.0125 & 0.0225 & 0.0275 \end{bmatrix}$$ - node reactance matrix,

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$ - branch-node connections matrix,

$$\mathbf{H} = \mathbf{diag}(\mathbf{b}_{\mathbf{branch}})\mathbf{CX} = \begin{bmatrix} 0 & 0.4 & -0.4 \\ 0 & 0.6 & 0.4 \\ 0 & -0.4 & -0.6 \\ 1 & 1 & 1 \end{bmatrix}$$ - power transfer matrix.

Particularly, we have before optimization the following form of variable vectors and matrices:

$$\mathbf{P} = \begin{bmatrix} \mathbf{P_x} \\ \mathbf{P_y} \end{bmatrix} = \begin{bmatrix} \mathbf{P_{gx}} - \mathbf{P_{dx}} \\ \mathbf{P_{gy}} - \mathbf{P_{dy}} \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$ - network nodes,

$\mathbf{P_{gx}} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$ and $\mathbf{P_{dx}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ - nodes with wind farms,

$\mathbf{P_{gx}} = [0]$ and $\mathbf{P_{dx}} = [-20]$ - node without wind farms,

$P_n = -5$ - reference node /slack bus,

$$\mathbf{S_{max}} = \begin{bmatrix} 20 \\ 5 \\ 5 \\ 10 \end{bmatrix}$$ - maximal branch flows,

$$\mathbf{P_b} = \mathbf{HP} = \begin{bmatrix} 0 & 0.4 & -0.4 \\ 0 & 0.6 & 0.4 \\ 0 & -0.4 & -0.6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 15 \\ -20 \end{bmatrix} = \begin{bmatrix} 14 \\ 1 \\ 6 \\ 5 \end{bmatrix}$$ - branch flows.

It is seen that the power flow at c-branch is higher than permissible.

For constraints fulfilment purpose an optimization in Matlab has been carried out with *linprog* function. The results are shown on Fig. 3.

To avoid lines overloading the point should be moved on system power balance line. It can be done in many ways, depending on chosen objective function. The goal of maximal total wind generation has been chosen in this example.

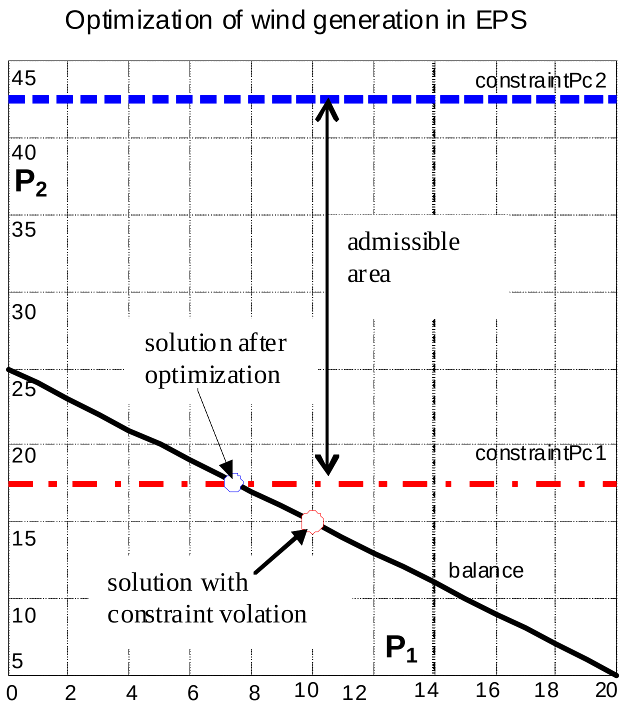

Figure 3. Wind generation before and after optimization.

The results of optimization are as follows:

$$\mathbf{x_{opt}} = \mathbf{P_{gxopt}} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 17.5 \end{bmatrix}$$ - optimal power of wind farms,

$$\mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 17.5 \\ -20 \end{bmatrix}$$ - optimal node powers,

$$\mathbf{P_b} = \mathbf{HP} = \begin{bmatrix} 0 & 0.4 & -0.4 \\ 0 & 0.6 & 0.4 \\ 0 & -0.4 & -0.6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 7.5 \\ 17.5 \\ -20 \end{bmatrix} = \begin{bmatrix} 15 \\ 2.5 \\ 5 \\ 5 \end{bmatrix}$$ - optimal branch flows.

IX. FINAL REMARKS

Wind generation affects power flow in the transmission network and may even change the branch power direction. Improper location of wind farm can effect bottleneck situations.

The transmission lines and transformers should not be overloaded after interconnecting wind farms into transmission network.

Wind generation results in decreasing conventional generation. However, the technical minimum of a thermal unit in power stations must not be violated. This is the most important obstacle for wind generation in power systems.

The assessment of bus wind generation can be formulated as the minimization of a linear goal function subjected to linear equality (generation and demand balance) and inequality constraints (permissible branch flows and the technical minimum of thermal unit generation).

It has been shown that the Matlab *linprog* function can be used to find the value of proper bus wind generation in a electric power system.

The mathematical background and example computation are presented in the paper.

Further investigations should be made, especially for various goal functions

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